PERPENDICULAR LINES: PROBLEM SOLVING

GRADE 9

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

P and Q are two points.

The coordinates of P are $(-1,6)$

The coordinates of Q are $(5,-4)$

Find an equation of the perpendicular bisector of PQ.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

MIDPOINT OF PQ $(2,1)$ M1

GRADIENT OF PQ $\frac{6--4}{-1-5} = -5/3$ M1

NEW GRADIENT $3/5$ M1

$1 = 3/5(2) + c$ M1

$c = -0.2$ A1

$y = 3/5x - 1/5$ A1

.....

(6 marks)

Work out the equation of the perpendicular bisector of $P(3,-1)$ and $Q(5,7)$.

Give your answer in the form $y = ax + b$

MIDPOINT OF PQ $(4, 3)$ M1

GRADIENT OF PQ $\frac{7 - -1}{5 - 3} = 4$ M1

NEW GRADIENT : $-\frac{1}{4}$ M1

$3 = -\frac{1}{4}(4) + c$ M1

$c = 4$

$y = -\frac{1}{4}x + 4$ A1

.....

(5 marks)

ABC is an isosceles triangle with $AB = AC$.

B is the point with coordinates $(-1,5)$
C is the point with coordinates $(2,10)$
M is the midpoint of BC.

$M = (0.5, 7.5)$ M1

Find an equation of the line through the points A and M.
Give your answer in the form $py + qx = r$ where p, q and r are integers to be found.

$\frac{10-5}{2--1} = \frac{5}{3}$ = GRADIENT OF BC M1

GRADIENT OF m : $-\frac{3}{5}$ M1

$7.5 = -\frac{3}{5}(0.5) + c$

$c = 7.8$ M1

$y = -\frac{3}{5}x + 7.8$

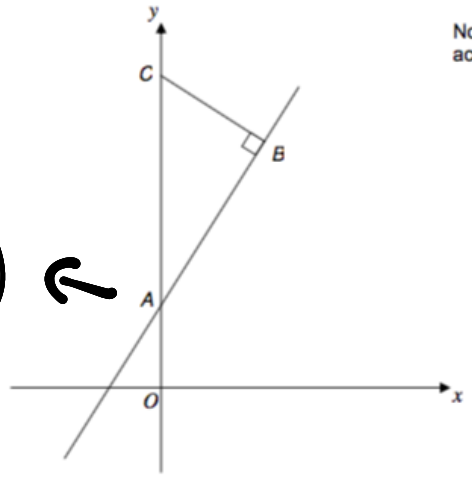
$5y + 3x = 39$ A1

.....

(5 marks)

On the diagram

- A and C are on the y-axis
- C is (0, 7)
- The equation of the line through A and B is $y = 2x + 1$
- BC is perpendicular to AB.



Not drawn accurately

B1 (0, 1) ←

∴ AC is 6

Work out the area of triangle ABC.

GRADIENT OF BC = $-\frac{1}{2}$ B1

∴ BC = $y = -\frac{1}{2}x + 7$

$2x + 1 = -\frac{1}{2}x + 7$ M1

∴ $x = 2.4$ m1

$\frac{1}{2} \times 6 \times 2.4 =$ m1

A1

7.2 units^2

.....

(6 marks)

The straight line L passes through point A (– 6, 2) and point B (5, 3)
The straight line M is perpendicular to L and passes through the midpoint of A and B.
The line M intersects the line $x = -1$ at point C.

Calculate the area of triangle ABC.

$(-\frac{1}{2}, \frac{5}{2})$ B1

$L: \frac{2-3}{-6-5} = \frac{1}{11}$

$\frac{5}{2} = -11\left(-\frac{1}{2}\right) + c \quad \therefore c = -3$ M1

$M: y = -11x - 3$

$y = -11(-1) + 8 = 8$ M1

$C = (-1, 8)$

$A(-6, 2)$

$B(5, 3)$

$\frac{1}{2} \times \sqrt{122} \times \frac{\sqrt{122}}{2}$

$= \frac{30 \cdot 5}{}$ A1

$AB = \sqrt{(3-2)^2 + (5--6)^2} = \sqrt{122}$ M1

PERPENDICULAR: $\sqrt{(8-\frac{5}{2})^2 + (-1--\frac{1}{2})^2}$ M1

$= \frac{\sqrt{122}}{2}$

.....

(6 marks)

Q6. AtoZrevision.com

The straight line L has equation $3x + 2y = 17$

The point A has coordinates $(0,2)$
The straight line M is perpendicular to L and passes through A.

Line L crosses the y-axis at the point B.
Lines L and M intersect at the point C.

Work out the area of triangle ABC.

$2y = -3x + 17$

$y = -\frac{3}{2}x + 8.5$ M1

GRADIENT of M: $\frac{2}{3}$

$2 = \frac{2}{3}(0) + c$

$c = 2$

$y = \frac{2}{3}x + 2$ M1

$y = -\frac{2}{3}(0) + 8.5$ M1

$B = (0, 8.5)$

$-\frac{3}{2}x + 8.5 = \frac{2}{3}x + 2$

$x = 3 \therefore y = 4$ M1

$A(0,2), B(0,8.5), C(3,4)$

$ABC = 9.75$ A1

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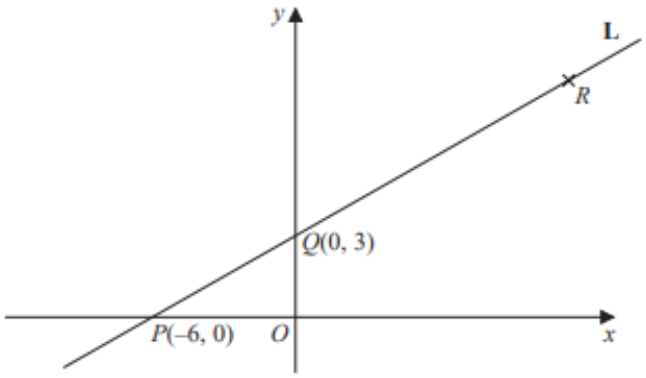
(5 marks)

Here is a sketch of the line **L**.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line **L**.
The point R is such that PQR is a straight line and $PQ:QR = 2:3$.

The coordinates of R are (9, 7.5).

Find an equation of the line that is perpendicular to **L** and passes through Q.

$PQ = y = mx + 3$ (m is $\frac{1}{2}$) B_1

$y = \frac{1}{2}x + 3$ m_1

$y = -2x + c$

$3 = -2(0) + c \quad \therefore c = 3$ m_1

$y = -2x + 3$ A_1

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(4 marks)

Line L has equation $4y - 6x = 33$

Line M goes through the point $A\ (5,6)$ and the point $B(-4, k)$

L is perpendicular to M.

Work out the value of k.

so $6/4 \rightarrow -2/3$ M1

$y = \frac{6}{4}x + \frac{33}{4}$ M1

$M: \frac{k-6}{-4-5} = -\frac{2}{3}$ M1

$k - 6 = \frac{18}{3} = 6$

$k - 6 = 6$

$k = 12$ A1

.....

(4 marks)

The line joining $(a, 7)$ to $(-1, 10)$ is perpendicular to the line joining $(a, 7)$ to $(9, -6)$.

Find the possible values of a.

$$\frac{10-7}{-1-a} \times \frac{-6-7}{9-a} = -1$$

m1 A1

$$\frac{3 \times -13}{(-1-a)(9-a)} = -1$$

m1

$$39 = -9 - 8a + a^2$$

$$a^2 - 8a - 48 = 0$$

m1

$$a = 12 \qquad a = -4$$

A1 A1

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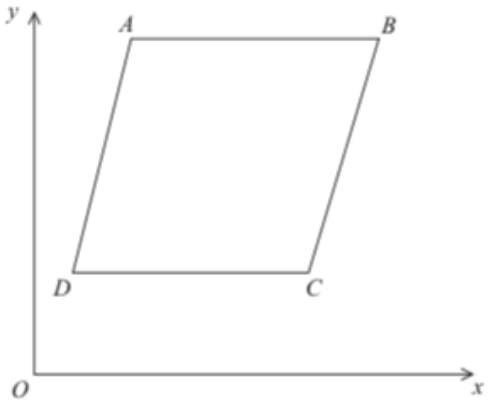
(6 marks)



$ABCD$ is a rhombus.

The coordinates of A are $(5,11)$

The equation of the diagonal DB is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal AC.

P1

AC IS PERPENDICULAR TO DB.

GRADIENT OF AC = -2 P1

$11 = -2(5) + c$ P1

$c = 21$

AC = $y = -2x + 21$ A1

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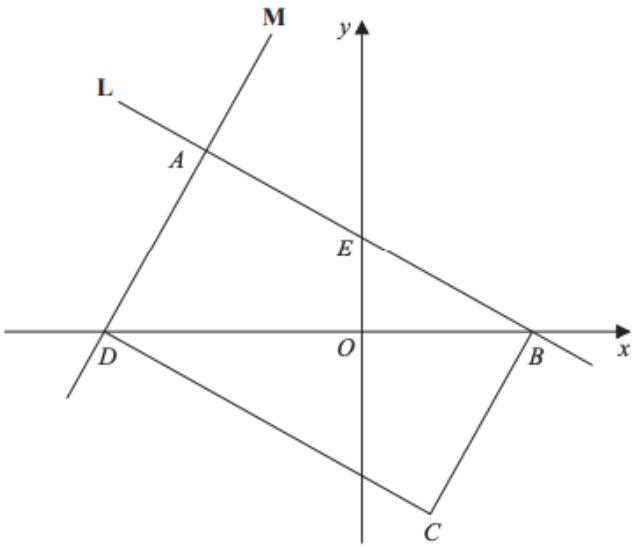
(4 marks)



$ABCD$ is a rectangle.

A, E and B are points on the straight line **L** with equation $x + 2y = 12$
A and D are points on the straight line **M**.

$AE = EB$

Find an equation for **M**.

$2y = -x + 12$

$y = -\frac{1}{2}x + 6$

GRADIENT : 2 P1

B: $0 = -\frac{1}{2}x + 6$

$-6 = -\frac{1}{2}x \quad \therefore \quad x = 12 \quad (12, 0)$ P1

E: $y = -\frac{1}{2}(0) + 6$ $(0, 6)$

P1

A : $(-12, 12)$

A1

$12 = 2(-12) + c$

$c = 36$

$y = 2x + 36$

(PHEW!)

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(4 marks)

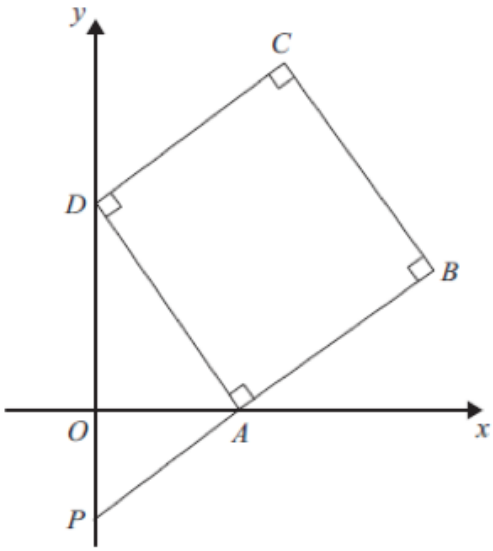

Diagram **NOT** accurately drawn

$ABCD$ is a square.
P and D are points on the y-axis.
A is a point on the x-axis.
PAB is a straight line.

The equation of the line that passes through the points A and D is $y = -2x + 6$.

Find the length of PD.

A: $-2x + 6 = 0 \therefore x = 3$
$(3, 0)$ M1

D: $y = -2(0) + 6 \therefore y = 6$
$(0, 6)$ M1

PB: $y = \frac{1}{2}x + c$
$0 = \frac{1}{2}(3) + c$
$c = -1.5$ A1
$P(0, -1.5) \therefore PD = 7.5$ M1

.....

(4 marks)

ABC is an isosceles triangle such that

$AB = AC$
A has coordinates $(4,37)$
B and C lie on the line with equation $3y = 2x + 12$

Find an equation of the line of symmetry of triangle ABC.
Give your answer in the form $px + qy = r$ where p, q and r are integers.

$y = \frac{2}{3}x + 4$ M1

PERP. GRADIENT $= -\frac{3}{2}$ M1

$37 = -\frac{3}{2}(4) + c \therefore c = 43$ A1

M1 $y = -\frac{3}{2}x + 43$

$2y = -3x + 86$

$3x + 2y = 86$ A1

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(5 marks)

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

Reflection in the line l transforms the point with coordinates $(5,3)$ into the point with coordinates $(1,-1)$.

What is the equation of the line l? Tick the correct answer.

L IS THE PERPENDICULAR BISECTOR.

MIDPOINT : $(3,1)$

GRADIENT : $\frac{4}{4} = 1$

$\therefore$ GRADIENT OF L $= -1$

L PASSES THROUGH $(3,1)$ $\therefore$ →

$y = x - 2$

$y = 1$

$x = 3$

$y = 2 - x$

$y = 4 - x$

Q2. AtoZrevision.com

[redacted]

DIAGONALS BISECT AT RIGHT ANGLES.

MIDPOINT OF AC $= (2,1)$ WHICH IS THE CENTRE OF THE SQUARE.

GRADIENT OF AC : $\frac{0-2}{4-0} = -\frac{1}{2}$

$\therefore$ BD $= 2$

$\therefore$ BD $= y - 1 = 2(x-2) = 2x - 3$