3D PYTHAGORAS

GRADE 8

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

A cuboid has length 3 cm, width 4 cm and height 12 cm.

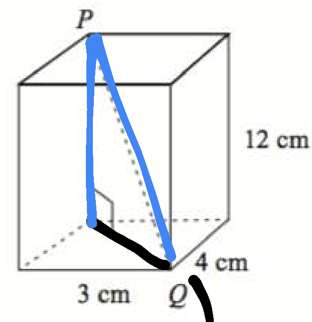


Diagram **NOT** accurately drawn

Work out the length of PQ.

$\sqrt{3^2 + 4^2} = \sqrt{25}$ M1

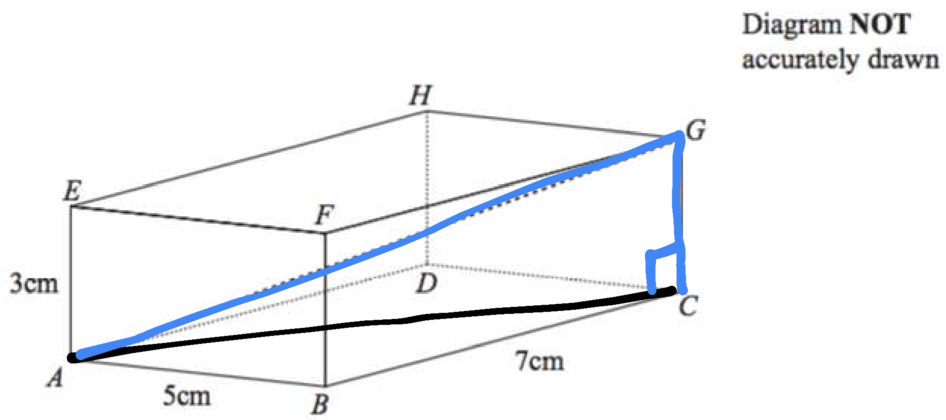
$= 5$

$\sqrt{12^2 + 5^2} = 13$ M1

$PQ = 13$ A1

.... cm

(3 marks)



The diagram shows a cuboid ABCDEFGH.

AB = 5cm

BC = 7cm

AE = 3cm

Calculate the length of AG.

Give your answer correct to 3 significant figures.

$$AC = \sqrt{5^2 + 7^2} = \sqrt{74}$$ M1

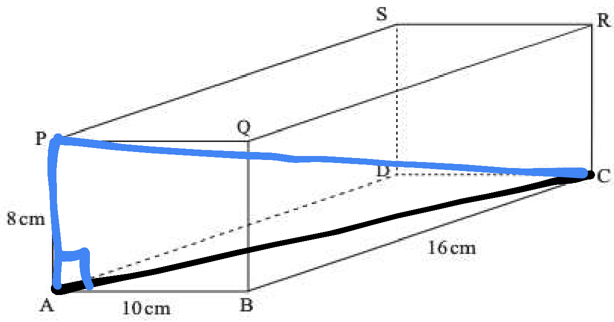
$$\sqrt{(\sqrt{74})^2 + 3^2} = \sqrt{83}$$ M1

$$AG = \sqrt{83} = 9.11\text{cm}$$ A1

....

(3 marks)

Calculate the length of the space diagonal PC of the cuboid.



$$AC = \sqrt{10^2 + 16^2}$$

$$= \sqrt{356}$$ M1

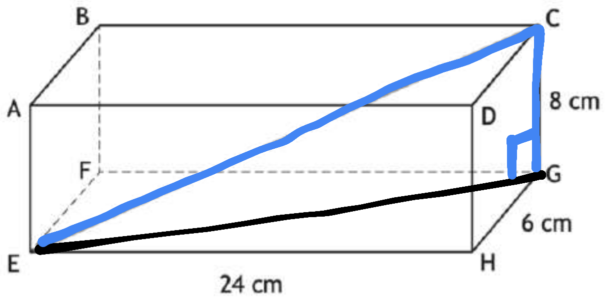
$$PC = \sqrt{(\sqrt{356})^2 + 8^2}$$ M1

$$= \sqrt{420} = 20.5\text{cm}$$ A1

....

(3 marks)

The diagram shows a cuboid, ABCDEFGH.



The length of the cuboid, EH, is 24 centimetres.

The breadth of the cuboid, HG, is 6 centimetres.

The height of the cuboid, CG, is 8 centimetres.

Calculate the length of EC, the space diagonal of the cuboid.

$EG = \sqrt{24^2 + 6^2} = \sqrt{612}$ M1

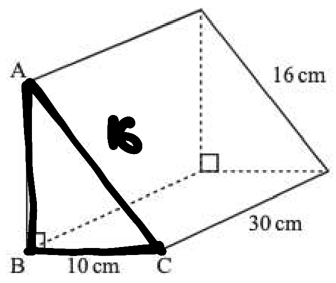
$EC = \sqrt{(\sqrt{612})^2 + 8^2}$ M1

$= 26\text{ cm}$ A1

....

(3 marks)

In the triangular prism ABC is a right-angled triangle. Calculate the length of AB.



$AB = \sqrt{16^2 - 10^2}$

M1 $= \sqrt{156}$

STRATEGY TO SOLVE

$= 12.5\text{ cm}$ A1

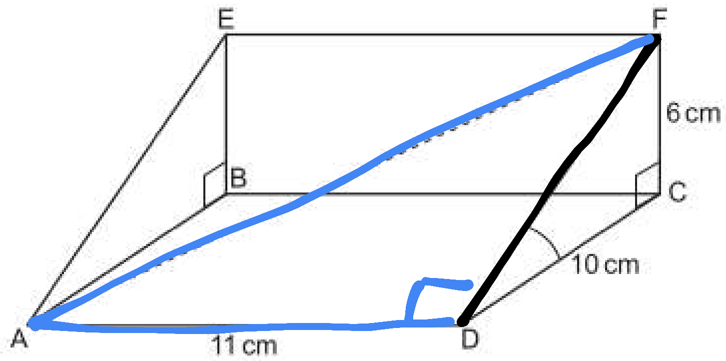
M1

A COMPLETE STRATEGY

....

(3 marks)

The diagram shows a right-angled triangular prism $ABCDEF$.



Length $AD = 11$ cm, length $CD = 10$ cm and length $CF = 6$ cm.

The size of angle $FDC = 31^{\circ}$.

Calculate the **exact** length of AF

$$DF = \sqrt{6^2 + 10^2} \quad M1$$

$$= \sqrt{136} \quad A1$$

$$AF = \sqrt{11^2 + (\sqrt{136})^2} \quad M1$$

$$= \sqrt{257} \quad A1$$

....

(4 marks)

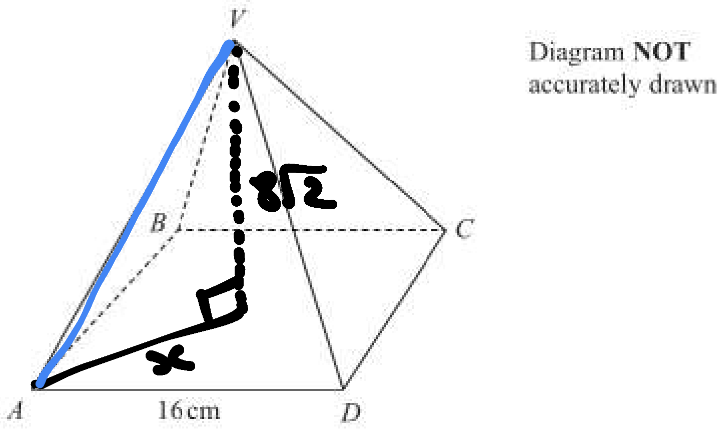


Figure 1

Figure 1 shows a right pyramid with vertex V and square base, $ABCD$, of side 16 cm.

The size of angle AVC is 90°

The perpendicular height of the pyramid is $8\sqrt{2}$ cm.

Find, in cm, the exact length of VA

$$AC = \sqrt{16^2 + 16^2} \quad M1$$
$$= \sqrt{512}$$
$$x = \tfrac{1}{2}\sqrt{512} = 8\sqrt{2} \quad A1$$
$$\sqrt{(8\sqrt{2})^2 + \left(\tfrac{1}{2}\sqrt{512}\right)^2} = 16 \quad A1$$

....

(3 marks)

The diagram shows a rectangular-based pyramid.

The length of the rectangular base is 4 cm and the width of the base is 3 cm.

Each slant edge of the pyramid is of length 6 cm.

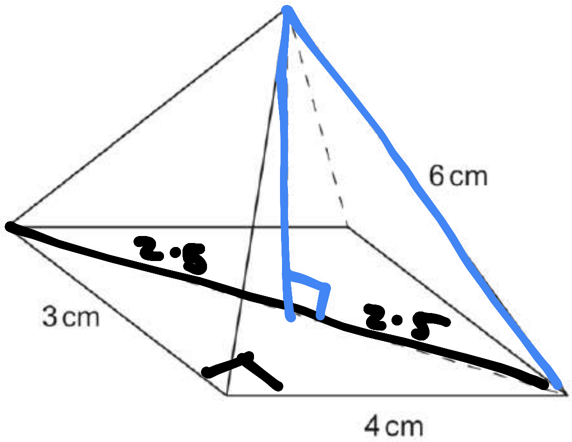


Diagram not drawn to scale

Calculate the perpendicular height of the pyramid.

$\sqrt{4^2 + 3^2} = 5$ M1

EACH HALF = 2.5 A1

$\sqrt{6^2 - 2.5^2} = 5.5$ A1

M1

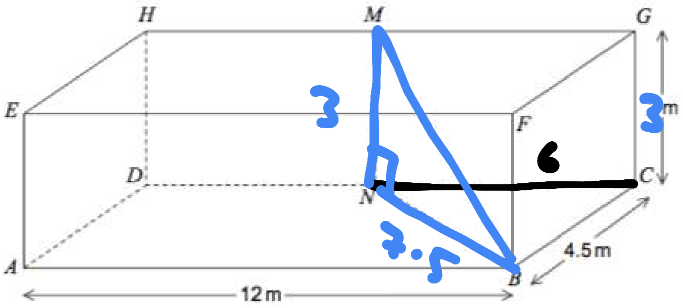
....

(4 marks)

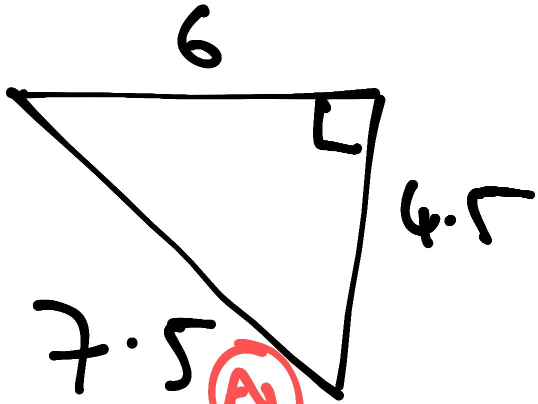
ABCDEFGH is a cuboid.

M is the midpoint of HG.

N is the midpoint of DC.



Find the length of MB



NB = $\sqrt{6^2+4.5^2} = 7.5$ M1 A1

MB = $\sqrt{3^2+7.5^2}$ = 8.07m M1 A1

....

(4 marks)

The cube below has an internal diagonal of length 20 cm.

Each edge of the cube is of length x cm.

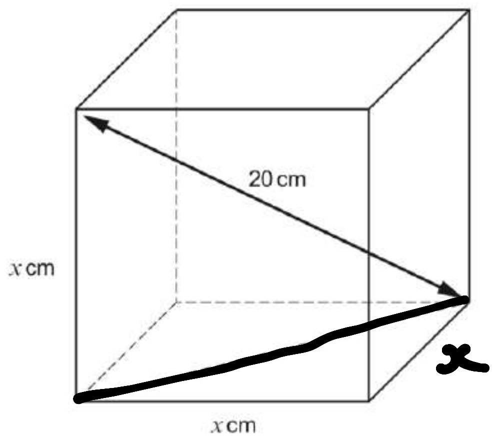


Diagram not drawn to scale

Calculate the value of x.

Give your answer correct to 1 decimal place.

$\sqrt{x^2 + x^2} = \sqrt{2x^2}$ M1 A1

$x^2 + \left(\sqrt{2x^2}\right)^2 = \cancel{20^2}\ 400$

$x^2 + 2x^2 = 400$ M1

$3x^2 = 400$

$x^2 = \frac{400}{3}$

$x = \sqrt{\frac{400}{3}} = 11.5\text{ cm}$ A1

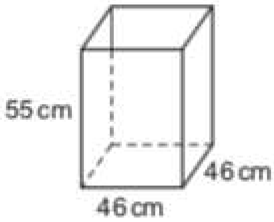
....

(4 marks)

Alvin has a crate in the shape of a cuboid.

The crate is open at the top.

The internal dimensions of the crate are 46 cm long by 46 cm wide by 55 cm high.



Alvin has a stick of length 95 cm.

Alvin places the stick in the crate so that the shortest possible length extends out above the top of the crate.

Calculate the length of the stick that extends out of the crate.

DIAGONAL (FLOOR):

$\sqrt{46^2+46^2} = \sqrt{4232}$

A1

M1

M1

$\sqrt{46^2+46^2+55^2} = 85.2$

$95 - 85.2 = 9.8$ A1

....

(4 marks)

The diagram shows a prism $ABCDEFGH$ with a horizontal base.

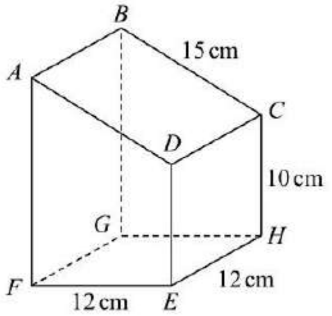


Diagram **NOT** accurately drawn

The base of the prism, $EFGH$, is a square of side 12 cm.

Trapezium $ADEF$ is a cross section of the prism where AF and DE are vertical edges.

$DE = CH = 10$ cm

$AD = BC = 15$ cm

Work out the length of BE.

Give your answer correct to one decimal place.

$BG = 10 + \sqrt{15^2 - 12^2}$ M1

$= 19$

$GE = \sqrt{12^2 + 12^2} = 16.97$ M1

$BE = \sqrt{19^2 + (16.97)^2}$ M1

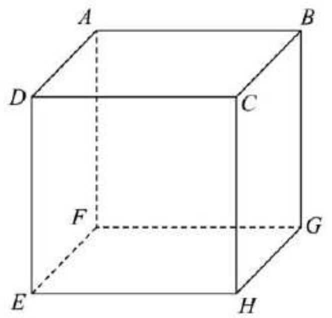
$= 25.5$

A1

....

(4 marks)

The diagram shows a cube.



$AH = 11.3$ cm correct to the nearest mm.

Calculate the lower bound for the length of an edge of the cube. $\rightarrow x$

11.25 M1 11.35

$11.25^2 = x^2 + x^2 + x^2$ M1

$11.25^2 = 3x^2$

$\sqrt{\frac{11.25}{3}} = 6.495$ A1

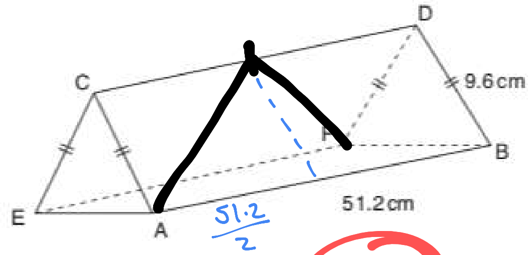
M1

ANSWER IN RANGE 6.49–6.50

....

(4 marks)

The diagram shows a prism, made from two isosceles triangles and three rectangles.



AC = CE, AB = 51.2 cm and BD = 9.6 cm.

A spider walks from A to the midpoint, M, of CD and then to F.

Calculate the shortest distance that the spider can walk from A to M to F.

Write your answer correct to 3 significant figures.

M1

$$\sqrt{\left(\frac{51.2}{2}\right)^2 + 9.6^2} = 27.34$$ A1

M1

$$27.34 \times 2 = 54.7$$ A1

3 s.f.

M1

....

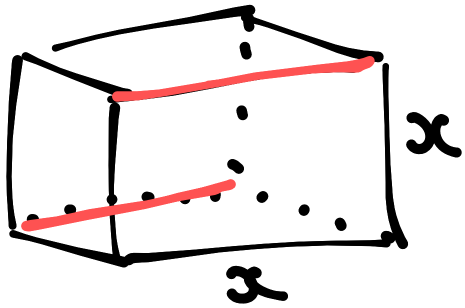
(5 marks)

Q1. AtoZrevision.com

A solid cube is divided into two pieces by a single rectangular cut. As a result, the total surface area increases by a fraction f of the surface area of the original cube.

What is the greatest possible value of f?

MUST CUT THE CUBE THROUGH THE RED LINES:

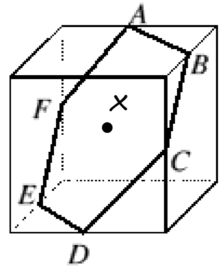


EXTRA SURFACE AREA FROM THIS CUT IS $2\sqrt{2}x$

$$\therefore f = \frac{2\sqrt{2}x}{6x^2} = \frac{\sqrt{2}}{3}$$

Q2.

A solid red plastic cube, volume 1 cm ^{3}, is painted white on its outside. The cube is cut by a plane passing through the mid-point of various edges, as shown.



What, in cm ^{2}, is the total red area exposed by the cut?

X = CENTRE

$$ABX = \sqrt{\underline{0.5}^2 + \underline{0.5}^2} = \frac{1}{\sqrt{2}}$$

XAB = EQUILATERAL

(X BC, XCD ALSO)

EACH AREA IS

$$\frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sin 60$$

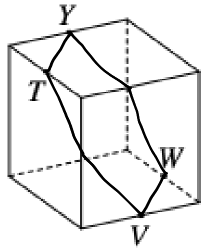
$$= \frac{1}{8}\sqrt{3}$$

AREA OF HEXAGON $= 6 \times \frac{1}{8}\sqrt{3}$

RED AREA IS DOUBLE THIS

$$\therefore \frac{3\sqrt{3}}{2}$$

The edge-length of the solid cube shown is 2. A single plane cut goes through the points Y, T, V and W which are midpoints of the edges of the cube, as shown.



What is the area of the cross-section?

THIS CREATES A HEXAGON.

SIDE LENGTH IS 2

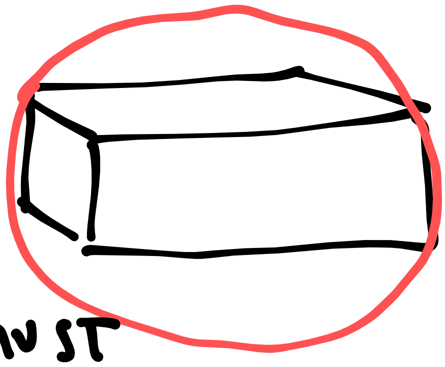
$\therefore$ DISTANCE BETWEEN MID-POINTS OF ADJACENT EDGES IS $\sqrt{2}$.

HEXAGON = 6 EQUILATERAL TRIANGLES AS IN PREVIOUS Q.

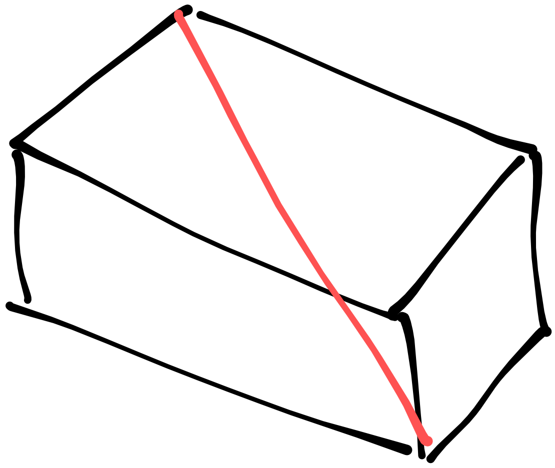
$$6 \times \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin 60$$

$$= 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

A cuboid has sides of lengths 22, 2 and 10. It is contained within a sphere of the smallest possible radius. What is the side-length of the largest cube that will fit inside the same sphere?



VERTICES MUST BE ON THE SPHERE.



RED LENGTH^2 $= 22^2 + 2^2 + 10^2$

$= 588$

RED LENGTH $= \sqrt{588}$

LET X BE THE SIDE LENGTH OF THE LARGEST CUBE THAT WILL FIT INSIDE THIS SPHERE.

$\therefore$ RED LENGTH $= (\sqrt{588})^2 = x^2 + x^2 + x^2$

$3x^2 = 588 \quad x^2 = 196 \quad \underline{\underline{x = 14}}$