3D TRIGONOMETRY

GRADE 8

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

Q1. AtoZrevision.com

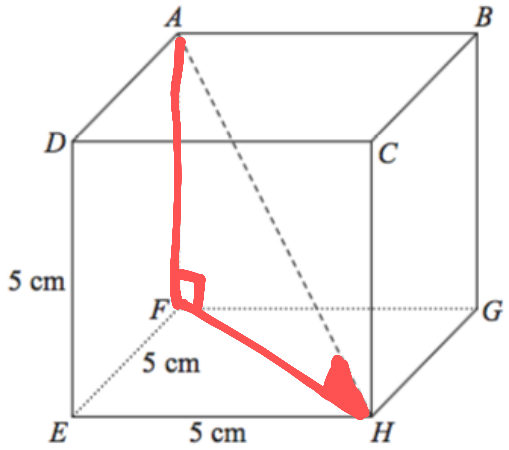


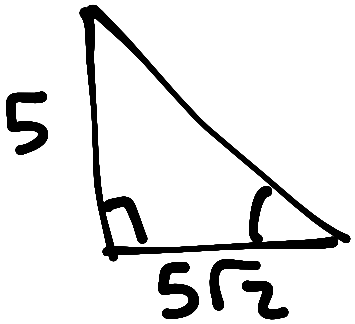
Diagram **NOT** accurately drawn

The diagram shows a cube *ABCDEFGH*.
The sides of the cube are of length 5 cm.

Calculate the size of the angle between the diagonal *AH* and the base *EFGH*.
Give your answer correct to 1 decimal place.

$FH = \sqrt{5^2 + 5^2}$ M1

$= 5\sqrt{2}$ A1



$\tan x = \frac{5}{5\sqrt{2}}$ M1

$\tan^{-1}\left(\frac{5}{5\sqrt{2}}\right) =$

35.264

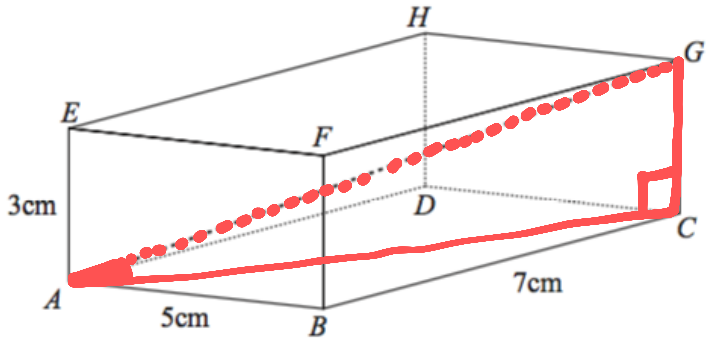
A1

35.3

.....

(4 marks)

Diagram **NOT** accurately drawn

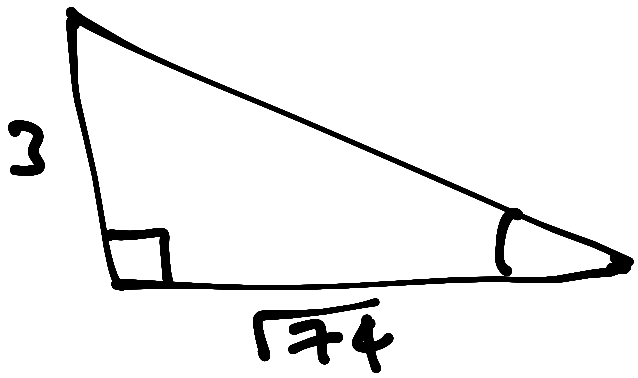


The diagram shows a cuboid ABCDEFGH.
AB = 5cm
BC = 7cm
AE = 3cm

Calculate the size of the angle between AG and the plane ABCD.
Give your answer correct to 1 decimal place.

$AC = \sqrt{5^2 + 7^2}$ M1

$= \sqrt{74}$ A1



$\tan^{-1}\left(\frac{3}{\sqrt{74}}\right) = 19.225$

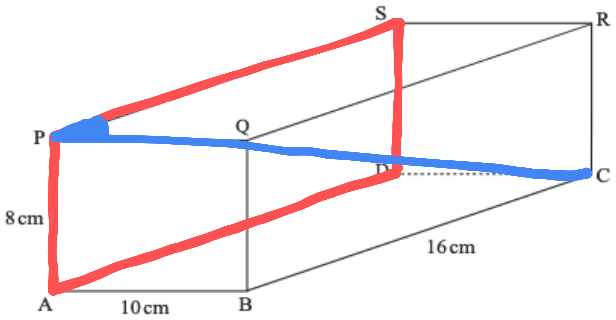
M1 A1

19.2

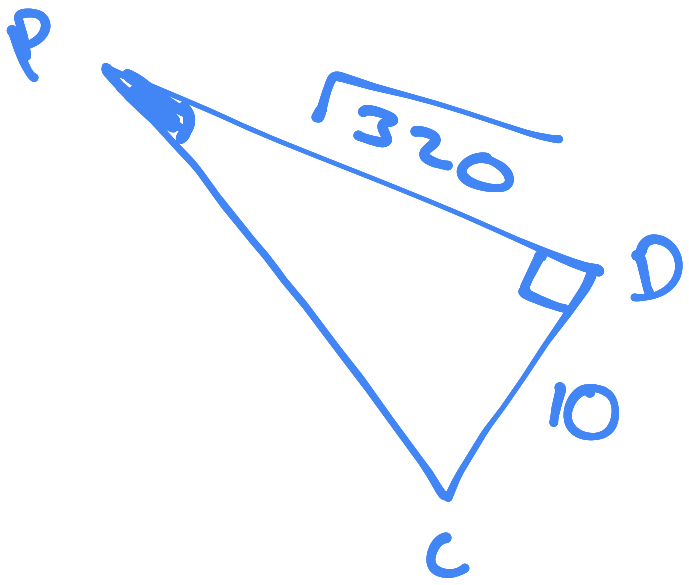
.....

(4 marks)

Here is a cuboid.



Find the angle between PC and the face PSDA.



$DP = \sqrt{16^2 + 8^2}$ M1

$= \sqrt{320}$

$\tan^{-1}\left(\frac{10}{\sqrt{320}}\right) = 19.2$ A1

M1

.....

(3 marks)

The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

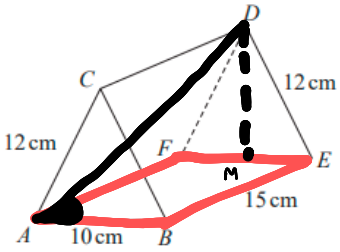
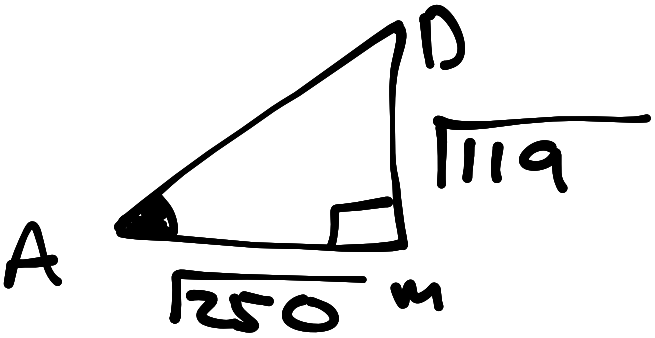


Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12$ cm
$AB = 10$ cm
$BE = 15$ cm

Calculate the size of the angle between AD and the base $ABEF$.
Give your answer correct to 3 significant figures.

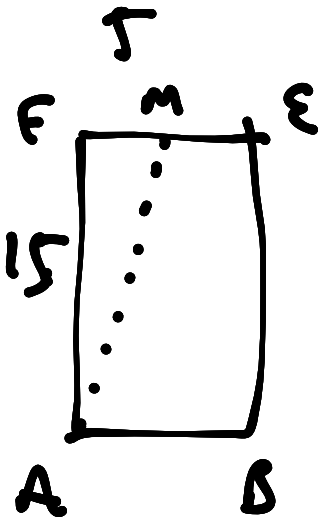


$DM = \sqrt{12^2 - 5^2}$

$= \sqrt{119}$ A1

$AM = \sqrt{15^2 + 5^2}$

$= \sqrt{250}$ A1



$\tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.6$ M1

34.6 A1

(4 marks)

The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.
AB = 10 cm. *BC* = 7 cm.
M is the midpoint of *AD*.
The vertex *T* is vertically above *M*.
MT = 6 cm.

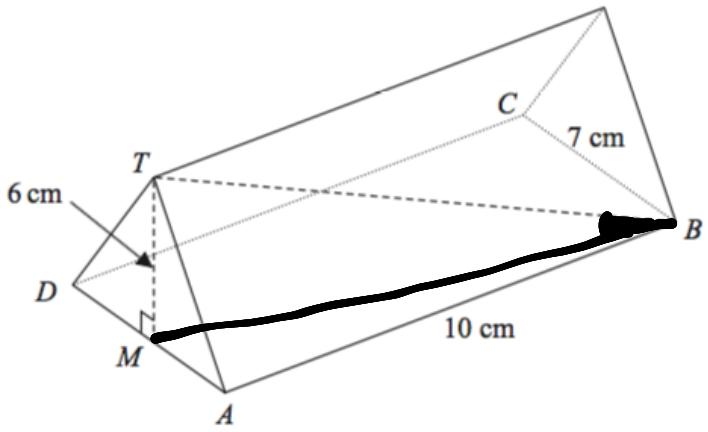
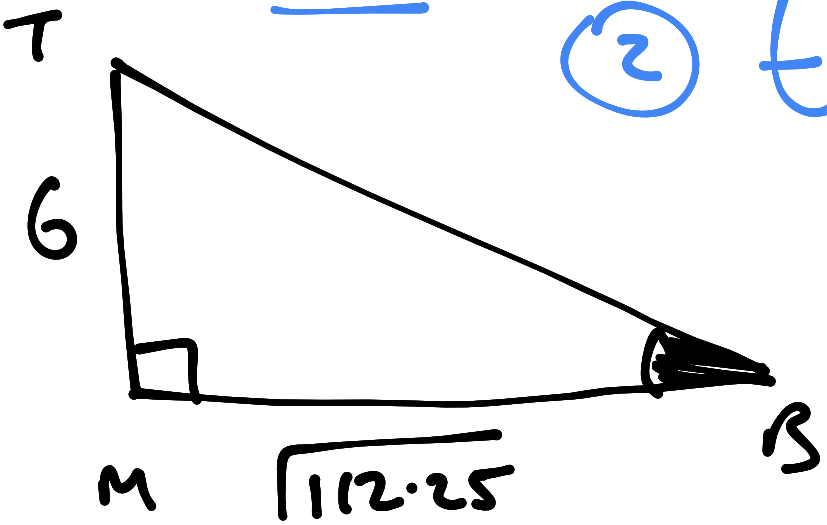


Diagram **NOT** accurately drawn

Calculate the size of the angle between *TB* and the base *ABCD*.
Give your answer correct to 1 decimal place.



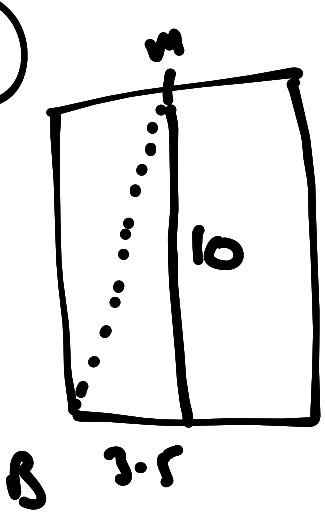
M1

$\tan^{-1}\left(\frac{6}{\sqrt{112.25}}\right) =$

(2)

29·52

(1)



$BM = \sqrt{10^2 + 3.5^2}$ M1

$= \sqrt{112.25}$

A1

A1

29.5

.....

(4 marks)

Q6. AtoZrevision.com

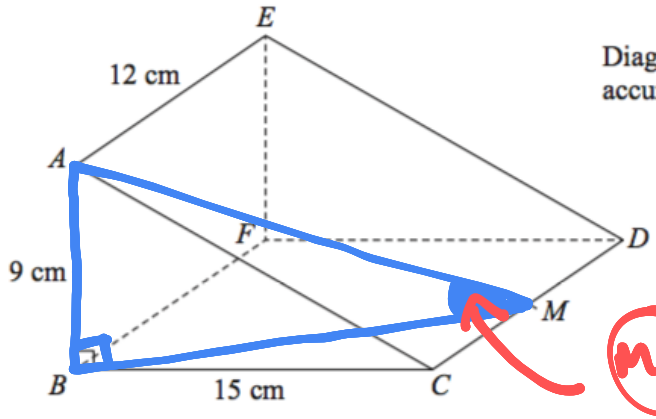
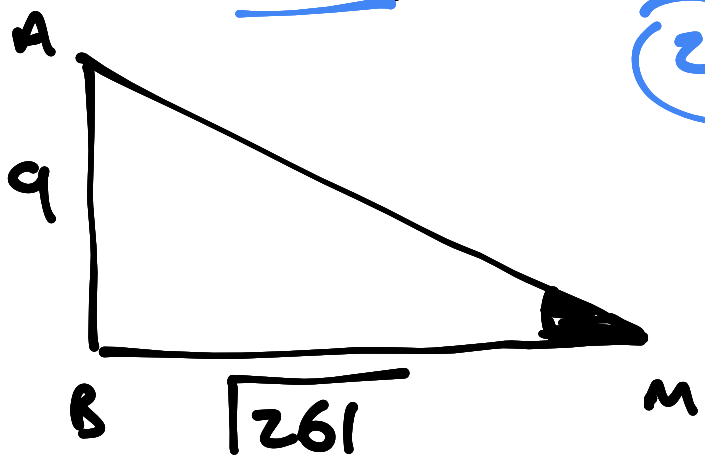


Diagram **NOT** accurately drawn

M1 IDENTIFYING ANGLE

ABCDEF is a triangular prism.
AB = 9 cm, *BC* = 15 cm and *AE* = 12 cm.
Angle *ABC* = 90°.
M is the midpoint of *CD*.

Calculate the size of the angle between *AM* and the plane *BCDF*.
Give your answer correct to 1 decimal place.

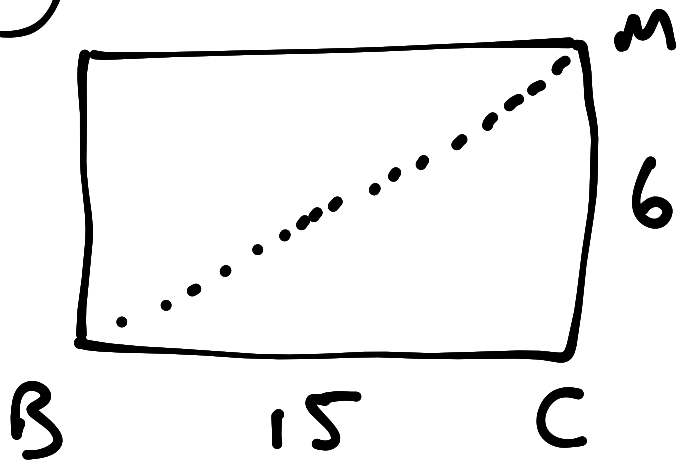


(2) $\tan^{-1}\left(\frac{9}{\sqrt{261}}\right) =$

29.12

M1

(1)



M1

$BM = \sqrt{15^2 + 6^2}$

$= \sqrt{261}$ A1

29.1

A1

(5 marks)

The diagram shows a prism.

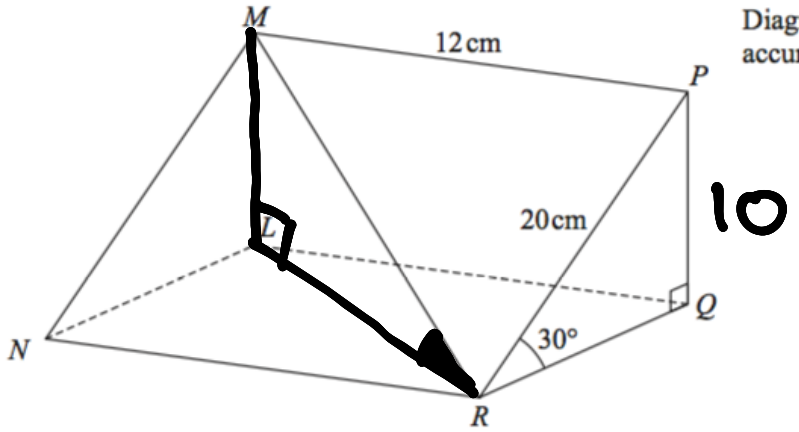


Diagram **NOT** accurately drawn

Triangle *PQR* is a cross section of the prism.

PR = 20 cm
MP = 12 cm
Angle *PRQ* = 30°
Angle *PQR* = 90°

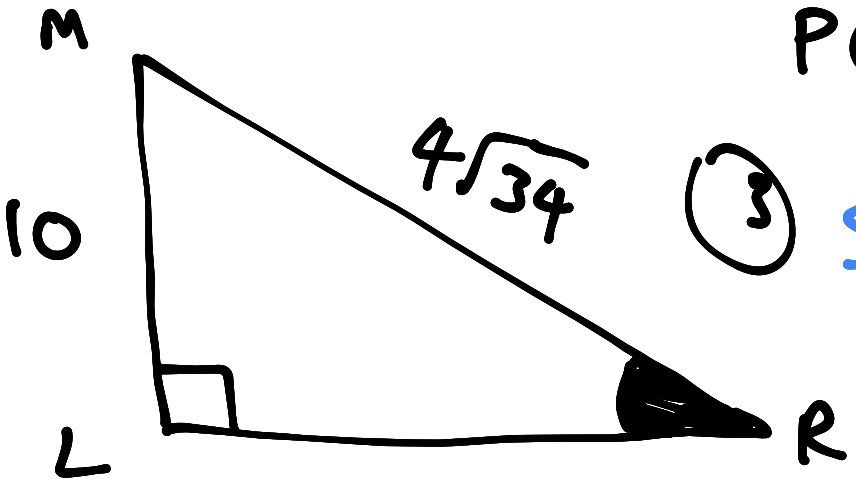
Calculate the size of the angle that the line *MR* makes with the plane *RQLN*.
Give your answer correct to 1 decimal place.

(1) $\sin(30) = \frac{PQ}{20}$ M1

$\sin(30) \times 20 = PQ$

$= 10$ A1

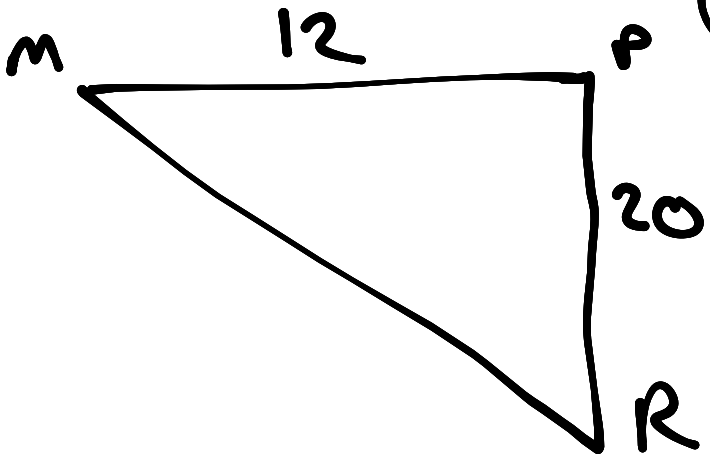
$PQ = ML$



(3) $\sin^{-1}\left(\frac{10}{4\sqrt{34}}\right) =$

25.4

(2)



$ML = \sqrt{12^2 + 20^2}$ M1

$= 4\sqrt{34}$ A1

25.4 A1

(5 marks)

The diagram shows a triangular prism.

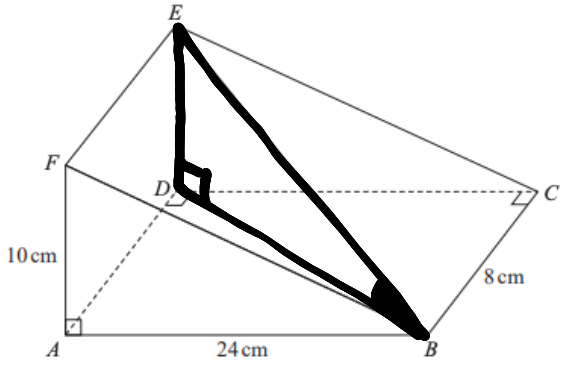
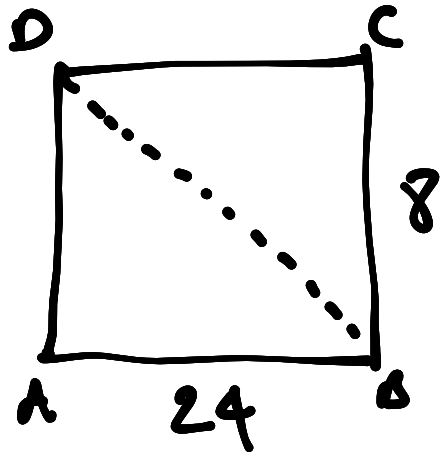
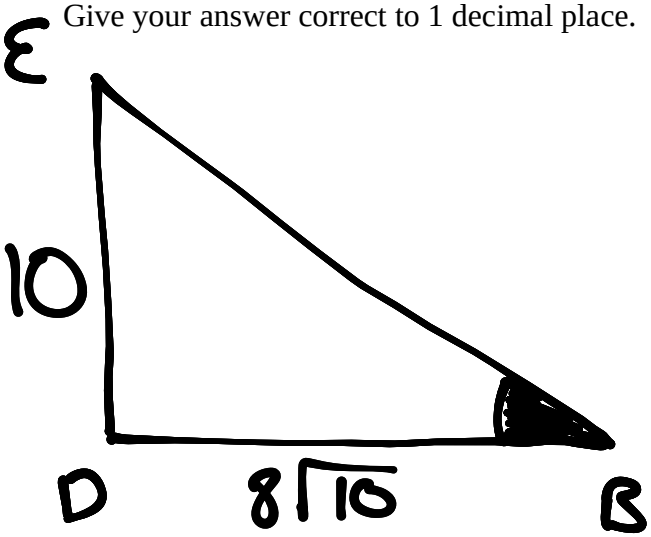


Diagram **NOT** accurately drawn



$AF = 10$ cm, $AB = 24$ cm and $BC = 8$ cm.
Angle FAB = angle ADC = angle $BCD = 90^\circ$

Work out the size of the angle between the line BE and the plane $ABCD$.
Give your answer correct to 1 decimal place.



$BD = \sqrt{24^2 + 8^2}$ M1

$= 8\sqrt{10}$

$\tan^{-1}\left(\frac{10}{8\sqrt{10}}\right) =$ M1

21.6

A1

21.6

.....

(3 marks)

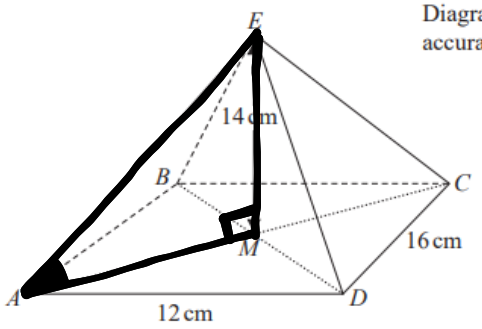


Diagram **NOT** accurately drawn

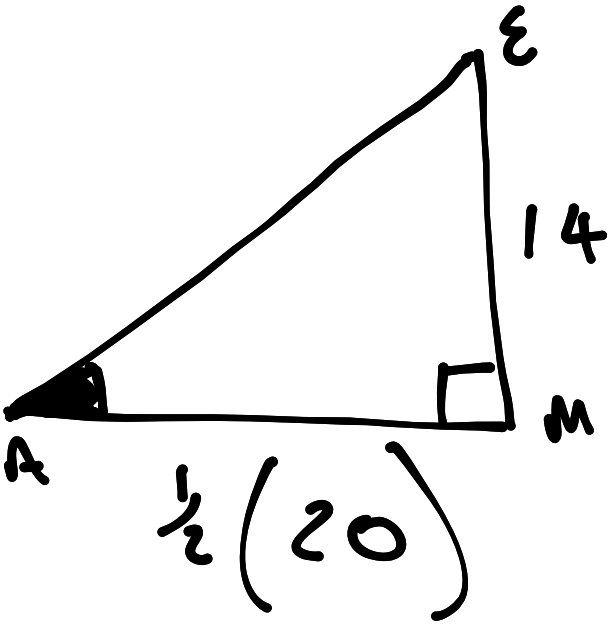
Figure 1

Figure 1 shows the right pyramid $ABCDE$. The base, $ABCD$, of the pyramid is a horizontal rectangle with $AD = 12$ cm and $CD = 16$ cm.

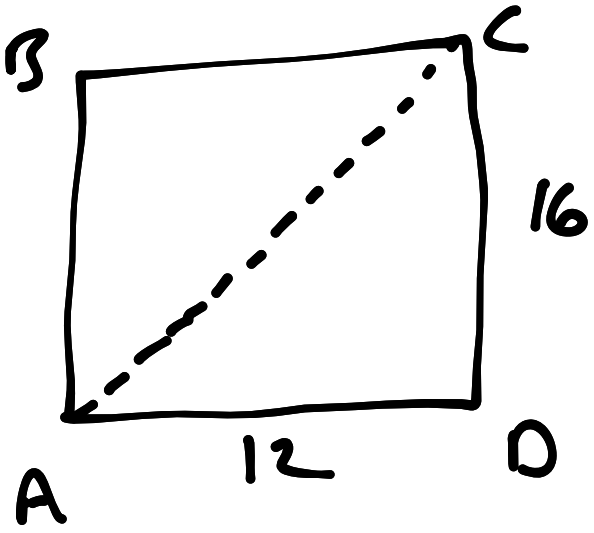
The height ME of the pyramid is 14 cm where M is the point of intersection of the diagonals of the base.

It can be shown that the sloping edges, EA, EB, EC and ED of the pyramid are all of equal length of 17.2 cm.

Calculate, in degrees to one decimal place, the size of the angle between AE and the base.



$\frac{1}{2}(20)$



$AC = \sqrt{16^2 + 12^2}$

$= 20$ M1

$\tan^{-1}\left(\frac{14}{10}\right) =$

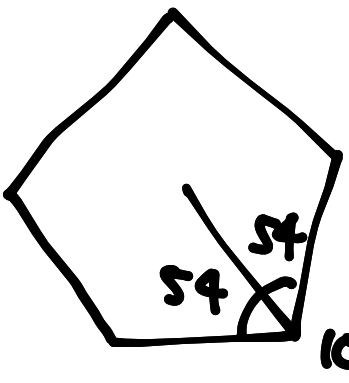
54.5

M1

A1 54.5

.....

(3 marks)



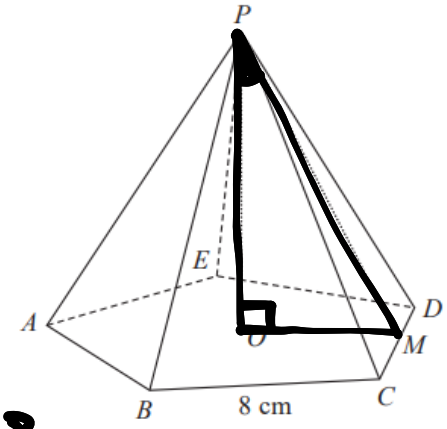
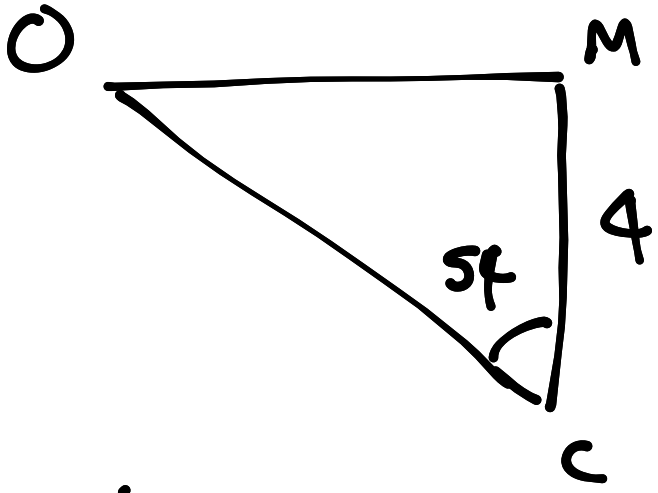


Diagram **NOT** accurately drawn

The diagram shows a pyramid with horizontal base $ABCDE$. $ABCDE$ is a regular pentagon, centre O and side 8cm.

The vertex P is vertically above O.
M is the midpoint of CD.
$OP = 10$cm.

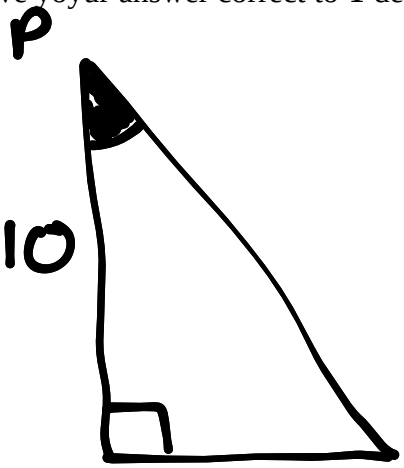
Calculate the size of angle APM.
Give yoyur answer correct to 1 decimal place.



$\tan 54 = \frac{OM}{4}$

$OM = 4 \times \tan 54$

$= 5.5055$ A1



$\tan^{-1}\left(\frac{5.5055}{10}\right) =$

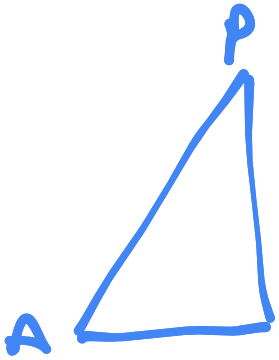
28.8 A1

$28.8 + 34.2$

$= 59.5$ A1

$AO = \frac{4}{\cos 54} = 6.8$ A1

$\angle APO = \tan^{-1}\left(\frac{6.8}{10}\right) = 34.2$ A1



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(5 marks)

The diagram shows a prism $ABCDEFGH$ with a horizontal base.

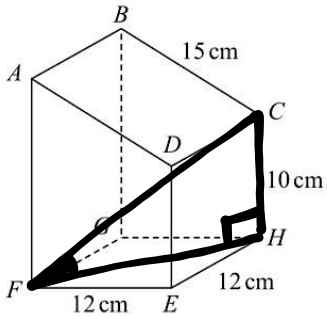


Diagram **NOT** accurately drawn

The base of the prism, $EFGH$, is a square of side 12 cm.
Trapezium $ADEF$ is a cross section of the prism where AF and DE are vertical edges.

$DE = CH = 10$ cm
$AD = BC = 15$ cm

Work out the size of the angle between CF and the base $EFGH$.
Give your answer correct to one decimal place.

$FH = \sqrt{12^2 + 12^2} = \sqrt{288}$ M1

$\tan^{-1}\left(\frac{10}{\sqrt{288}}\right)$ M1

$= 30.5$ 1 d.p. A1

.....

(3 marks)

The diagram shows a solid prism $ABCDEFGH$.

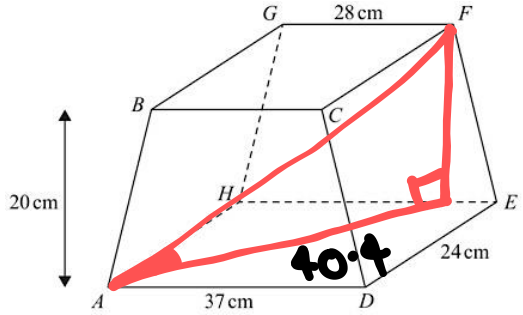


Diagram **NOT** accurately drawn

$\frac{37-28}{2} = 4.5$

The trapezium $ABCD$, in which AD is parallel to BC, is a cross section of the prism.
The base $ADEH$ of the prism is a horizontal plane.
$ADEH$ and $BCFG$ are rectangles.
The midpoint of BC is vertically above the midpoint of AD so that $BA = CD$.

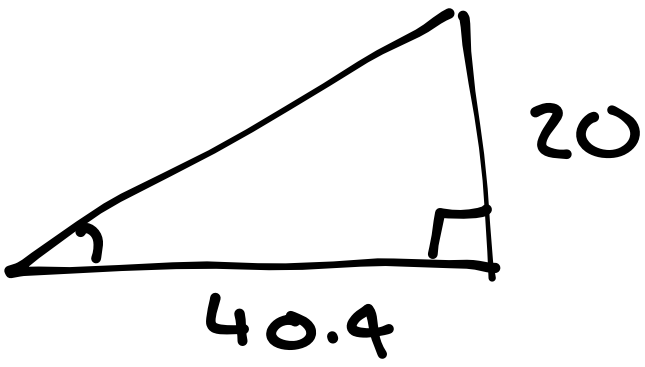
$AD = 37$ cm $\quad GF = 28$ cm $\quad DE = 24$ cm

The perpendicular distance between edges AD and BC is 20 cm.

Calculate the size of the angle between AF and the plane $ADEH$.
Give your answer correct to one decimal place.

$AF = \sqrt{24^2 + (37-4.5)^2}$ M1

$= 40.4$



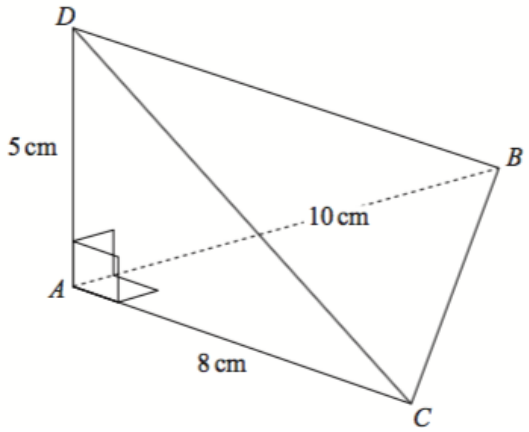
M1 $\tan^{-1}\left(\frac{20}{40.4}\right)$

$= 26.3$ A1

.....

(3 marks)

Diagram **NOT** accurately drawn

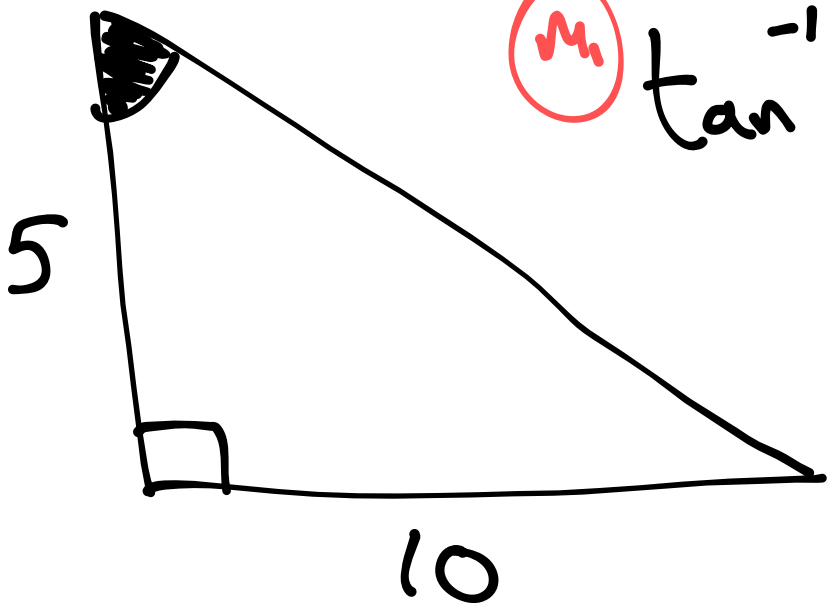


The diagram shows a tetrahedron.

AD is perpendicular to both AB and AC.
AB = 10 cm.
AC = 8 cm.
AD = 5 cm.
Angle BAC = 90°.

Calculate the size of angle BDC.
Give your answer correct to 1 decimal place.

$DB^2 = 5^2 + 10^2 = \sqrt{125}$



M1 $\tan^{-1}\left(\frac{10}{5}\right) =$

63.43

A1

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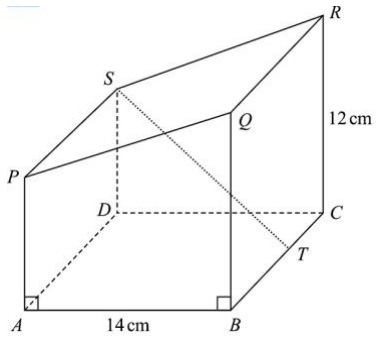
(2 marks)

Here is a prism $ABCDSPQR$.



The base $ABCD$ of the prism is a square of side 14 cm
T is the point on BC such that $BT:TC = 4:3$

The cross section of the prism is in the shape of a trapezium of area 147 cm ^{2}. $CR = 12$ cm

Find the size of the angle between the line ST and the base $ABCD$.
Give your answer correct to 1 decimal place.

$TC = 14 \times \frac{3}{3+4} = 6$ P1

$TD = \sqrt{14^2 + 6^2}$ P1

$= 2\sqrt{58}$

$SD: \quad 147 = \frac{1}{2}(SD+12) \times 14$ P1

$SD = 9$

$\tan^{-1}\left(\frac{9}{2\sqrt{58}}\right) =$ P1

P1

30.6

(5 marks)

The diagram shows a solid pyramid $ABCDE$ with a horizontal base.

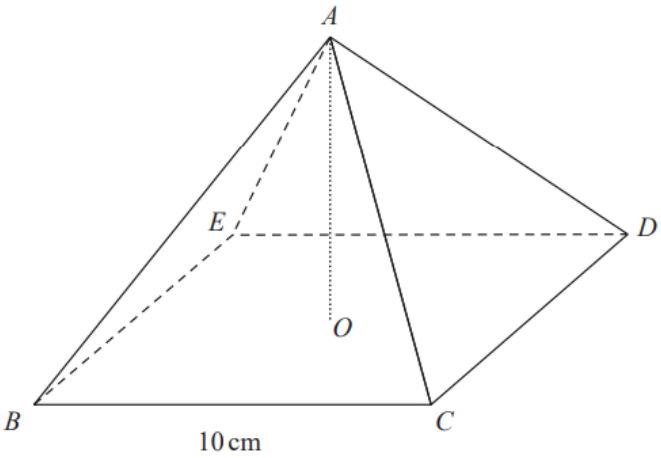


Diagram **NOT** accurately drawn

The base, $BCDE$, of the pyramid is a square of side 10 cm.

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$

The **total** surface area of the pyramid is 360 cm ^{2}.

Work out the size of the angle between AC and the base $ABCDE$.
Give your answer correct to 3 significant figures.

$360 = (10 \times 10) + 4 \times 0.5 \times 10 \times h$ M1

$360 = 104 \times 5 \times h$

$h = 13$

$AC = \sqrt{13^2 + 5^2} = \sqrt{194}$ M1

$AO = \sqrt{13^2 - 5^2} = 12$ A1

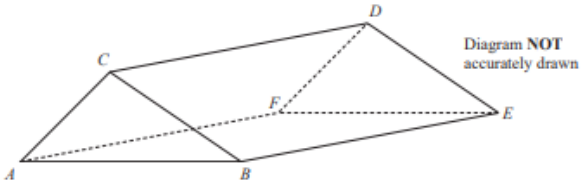
$OC = \sqrt{10^2 + 10^2} \div 2 = 5\sqrt{2}$ A1

$EC = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ A1

M1 $\tan^{-1}\left(\frac{AO}{OC}\right) = \tan^{-1}\left(\frac{12}{5\sqrt{2}}\right) =$ 59.5°

(6 marks)

The diagram shows the prism $ABCDEF$ with cross section triangle ABC.



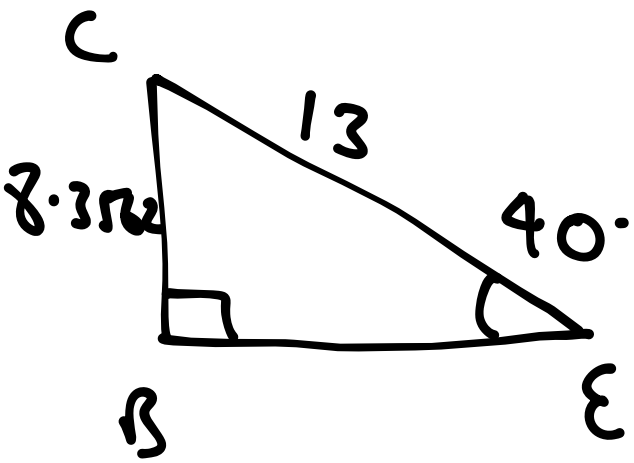
Angle $BEC = 40^\circ$ and angle ACB is obtuse.
$AC = 6$ cm and $CE = 13$cm

The area of triangle ABC is 22 cm ^{2}.

Calculate the length of AB.
Give your answer correct to one decimal place.

$CB = 13 \times \sin 40 = 8.3562$

M1



M1

$\frac{1}{2} \times 6 \times 8.3562 \times \sin ACB = 22$

$ACB = \sin^{-1}\left(\frac{22}{\frac{1}{2} \times 6 \times 8.3562}\right) = 61.35$

M1

$ACB = 118.647$

M1

A1

12.4

$AB^2 = 6^2 + 8.3562^2 - 2 \times 6 \times 8.3562 \times \cos 118.647 =$

M1

(6 marks)

The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

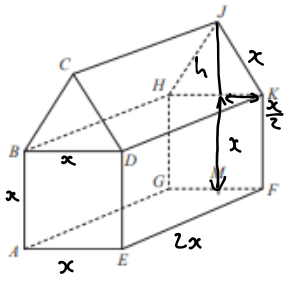


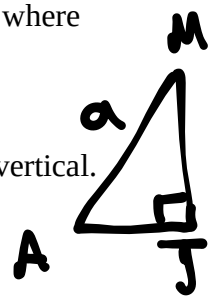
Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
BCD is an equilateral triangle
$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.
Angle $MAJ = y^\circ$

Given that $\tan y^\circ = T$

Find the value of T, giving your answer in the form $\frac{\sqrt{p}+\sqrt{q}}{17}$ where p and q are integers.

$$\frac{OPP}{MJ} = x + \frac{\sqrt{3}}{2}x$$



$$\frac{ADJ}{a^2} = (2x)^2 + \left(\frac{x}{2}\right)^2$$

$$a^2 = 4x^2 + \frac{x^2}{4} \quad M_1$$

$$h^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$h^2 = x^2 - \frac{x^2}{4}$$

$$h^2 = \frac{3x^2}{4}$$

$$h = \frac{\sqrt{3}}{2}x \quad M_1$$

$$a^2 = \frac{17x^2}{4}$$

$$a = \frac{\sqrt{17}}{2}x \quad A_1$$

$$\tan y = \frac{x + \frac{\sqrt{3}}{2}x}{\frac{\sqrt{17}}{2}x} \quad M_1$$

$$= \frac{1 + \frac{\sqrt{3}}{2}}{\frac{\sqrt{17}}{2}} = \frac{2+\sqrt{3}}{\sqrt{17}}$$

A_1

RATIONALISE:

$$\frac{2+\sqrt{3}}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}} = \frac{2\sqrt{17} + \sqrt{3}\sqrt{17}}{17}$$

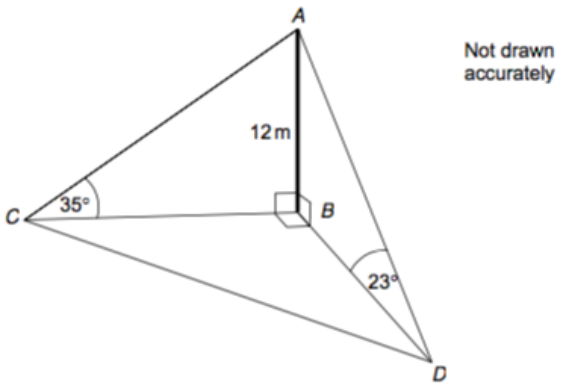
.....

(5 marks)

The diagram shows a vertical mast, AB, 12 metres high.
Points B, C and D are on a horizontal plane.

Point C is due West of B.
The angle of elevation of A from C is 35 °.

Point D is due South of B.
The angle of elevation of A from D is 23 °.



Calculate the distance CD.

$\tan 35 = \frac{12}{BC}$ M1

$BC = 17.137$ M1

$\tan 23 = \frac{12}{BD}$ M1

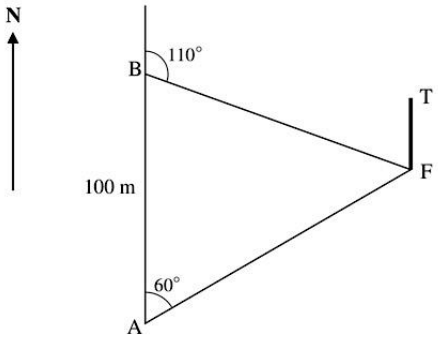
$BD = 28.2702$ M1

$\sqrt{BC^2 + BD^2} = 33.05$ M1 A1

33.1

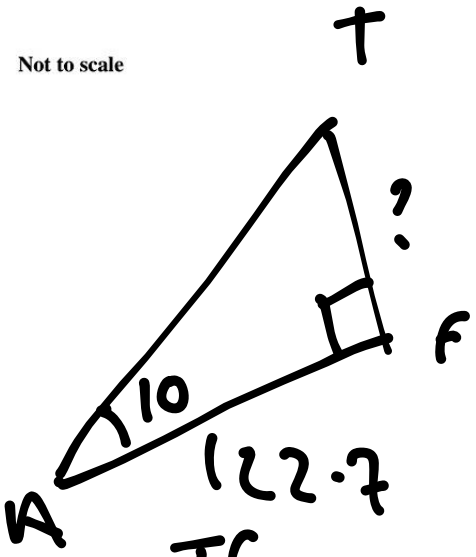
(6 marks)

Michael is at a point A and the base of a church tower is at a point F, as shown in Fig. 11.
He measures the bearing of the tower to be 060 °.
Michael walks 100 metres due North to the point B from where he measures the bearing of F to be 110 °.
The triangle ABF is in the horizontal plane.



Not to scale

Fig. 11



AF has been calculated to be 122.7 m.
BF has been calculated to be 113.1 m.
The angle of elevation of the top of the tower, T, from A is 10 °.
The height of the tower is 21.6 m.

C is the point on AB that is nearest to F.

Find CF and the angle of elevation from C to the top of the tower, correct to 1 decimal place.

$\tan 10 = \frac{TF}{122.7}$

$TF = 21.63$ M1

$CF = 122.7 \times \sin 60$ M1

$= 106.3$ A1

AOE: $\tan\theta = \frac{21.63}{106.3}$ M1

$\theta = 11.5°$ A1

.....

(5 marks)

The diagram shows cuboid $ABCDEFGH$.

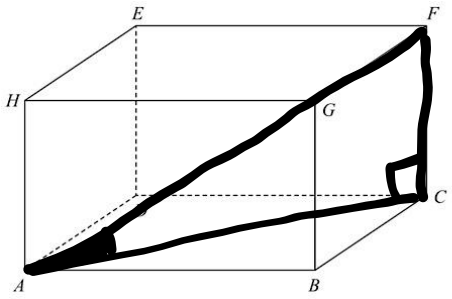


Diagram **NOT** accurately drawn

For this cuboid...

the length of AB : the length of BC : the length of CF = 4: 2: 3

Calculate the size of the angle between AF and the plane $ABCD$.
Give your answer correct to one decimal place.

$AC = \sqrt{(4)^2 + (2)^2} = \sqrt{20}$

$AF = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{29}$

M1

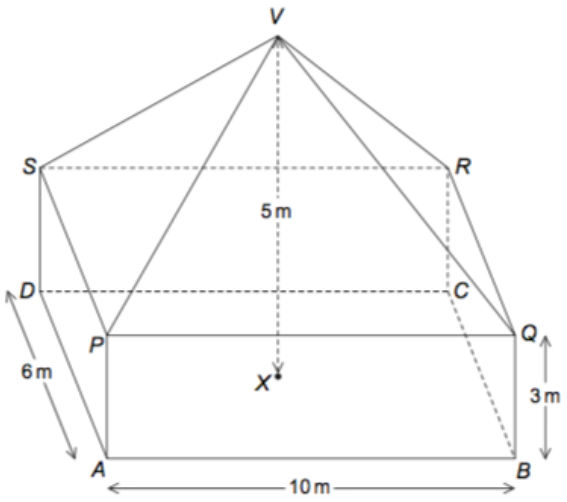
$CAF = \cos^{-1}\left(\frac{\sqrt{20}}{\sqrt{29}}\right) = 33.854$

M1

A1 33.9

(3 marks)

The diagram shows a cuboid ABCDPQRS and a pyramid PQRSV.
V is directly above the centre, X, of ABCD.



The total height, VX, is 5 metres.

(a) Work out the angle between the line VA and the plane ABCD.

$AC = \sqrt{10^2 + 6^2} = \sqrt{136}$ M1

$AX = \frac{\sqrt{136}}{2}$ M1

$\tan^{-1}\left(\frac{5}{\frac{\sqrt{136}}{2}}\right) = 40.5$ M1 A1

.....

(4)

(b) Work out the angle between the planes VQR and PQRS.

$\tan^{-1}\left(\frac{2}{5}\right) = 21.8$ M1 A1

.....

(2)