ALGEBRAIC PROOF

GRADE 8

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don't forget to have fun.
- Check your answers at the end.

Q1. AtoZrevision.com

Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \text{ is an even number}$$

for all positive integer values of n.

(3 marks)

Q2. AtoZrevision.com

(a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct? You must give reasons for your answer.

....

....

....

(1)

Q3. AtoZrevision.com

Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(3 marks)

Q4. AtoZrevision.com

n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(4 marks)

Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24

(6 marks)

Q6. AtoZrevision.com

An expression for the *n*th term of the sequence of triangular numbers is:

$$\frac{n(n+1)}{2}$$

Prove that the sum of any two consecutive triangular numbers is a square number.

(3 marks)

Q7. AtoZrevision.com

n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

(2 marks)

Q8. AtoZrevision.com

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(2 marks)

Q9. AtoZrevision.com

Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(3 marks)

Q10. AtoZrevision.com

Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(3 marks)

Q11. AtoZrevision.com

Prove algebraically that, for any three consecutive even numbers, the sum of the squares of the smallest even number and the largest even number is 8 more than twice the square of the middle even number.

(3 marks)

Q12. AtoZrevision.com

Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2. Show clear algebraic working.

(3 marks)

Q13. AtoZrevision.com

Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(4 marks)

Q14. AtoZrevision.com

For any three consecutive whole numbers, prove algebraically that the largest number and the smallest number are factors of the number that is one less than the square of the middle number.

(3 marks)

Q15. AtoZrevision.com

*****Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(4 marks)

Q16. AtoZrevision.com

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(3 marks)

Q17. AtoZrevision.com

Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12.

(3 marks)

Q18. AtoZrevision.com

Prove algebraically that the difference between any two different odd numbers is an even number.

(3 marks)

Q19. AtoZrevision.com

a, b, c are positive integers such that $a > b > c$

N is the largest three digit number that has the digits a, b and c.
K is the smallest three digit number that has the digits a, b and c.

(a) Use algebra to show that the difference between N and K is always a multiple of 99

(3)

(b) If $a > b$ and $b = c$ will the difference between N and K still be a multiple of 99?

Justify your answer.

(1)