

FACTORISING QUADRATICS: COMPLETING THE SQUARE

GRADE 8



Examination practice questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- If the number of marks for two similar questions isn't the same, this is likely due to them being modelled on different specifications. In this case, it is worth considering both mark schemes.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions will generally get more challenging as the document progresses. Some of the latter questions may go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don't forget to have fun.
- Check your answers at the end.

**Completing the square for an expression in the form ax^2+bx+c
where $a = 1$**

Q1.

AtoZrevision.com

$$x^2 + 2x + 3 \equiv (x + a)^2 + b$$

Find the values of the constants a and b .

$$\begin{aligned} & \textcircled{B1} \left[(x+1)^2 - 1 + 3 \right] \textcircled{M1} \\ & (x+1)^2 + 2 \textcircled{M1} \end{aligned}$$

$$a = \dots\dots\dots 1 \quad b = \dots\dots\dots 2$$

(3 marks)

Q2.

AtoZrevision.com

Show that $x^2 + 6x + 11$ can be written as $(x + p)^2 + q$, where p and q are integers to be found.

$$\begin{aligned} & \textcircled{B1} \left[(x+3)^2 - 9 + 11 \right] \textcircled{B1} \\ & (x+3)^2 + 2 \textcircled{B1} \end{aligned}$$

.....
(3 marks)

Q3.

AtoZrevision.com

Show that $x^2 - 4x + 15$ can be written as $(x + p)^2 + q$ for all values of x .

State the values of p and q .

$$\begin{aligned} & \left[(x-2)^2 - 4 + 15 \right] \text{ (M1)} \\ & \quad \text{(B1)} \\ & (x-2)^2 + 11 \\ & \quad \text{(A1)} \end{aligned}$$

$$p = -2 \quad q = 11$$

(3 marks)

Q4.

AtoZrevision.com

Express $x^2 - 10x + 40$ in the form $(x + a)^2 + b$, where a and b are integers to be found.

$$\begin{aligned} & \left[(x-5)^2 - 25 + 40 \right] \text{ (M1)} \\ & \quad \text{(B1)} \\ & (x-5)^2 + 15 \\ & \quad \text{(A1)} \end{aligned}$$

$$a = -5 \quad b = 15$$

(3 marks)

Q5.

AtoZrevision.com

Express $x^2 - 12x + 1$ in the form $(x - p)^2 + q$.

$$\left[(x-6)^2 - 36 + 1 \right] \quad \text{M1}$$

B1

$$(x-6)^2 - 35 \quad \text{A1}$$

.....
(3 marks)

Q6.

AtoZrevision.com

Write $x^2 - 10x + 22$ in the form $(x - a)^2 - b$.

$$\left[(x-5)^2 - 25 + 22 \right] \quad \text{M1}$$

B1

$$(x-5)^2 - 3 \quad \text{A1}$$

.....
(3 marks)

Q7.

AtoZrevision.com

Write $x^2 - 2x - 10$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

$$\begin{aligned} & \left[(x-1)^2 - 1 - 10 \right] \text{ (M1)} \\ & \quad \quad \quad \text{(B1)} \\ & (x-1)^2 - 11 \text{ (A1)} \end{aligned}$$

.....
(3 marks)

Q8.

AtoZrevision.com

Write $x^2 - 6x + 10$ in the form $(x - a)^2 + b$ where a and b are integers.

$$\begin{aligned} & \left[(x-3)^2 - 9 + 10 \right] \text{ (M1)} \\ & \quad \quad \quad \text{(B1)} \\ & (x-3)^2 + 1 \text{ (A1)} \end{aligned}$$

.....
(3 marks)

Q9.

AtoZrevision.com

Write the expression $x^2 - 4x - 3$ in the form $(x - a)^2 - b$.

$$\left[(x-2)^2 - 4 - 3 \right] \quad (M_1)$$

$$(x-2)^2 - 7 \quad (A_1)$$

.....
(3 marks)

Q10.

AtoZrevision.com

A function $f(x)$ is defined by $f(x) = x^2 - x + 4$

Use the method of completing the square to rewrite $f(x)$ in the form $(x + a)^2 + b$ where a and b are constants.

$$\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 4 \right] \quad (M_1)$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{15}{4} \quad (A_1)$$

.....
(3 marks)

Express $x^2 - 5x + \frac{1}{4}$ in the form $(x - a)^2 - b$.

$$\textcircled{B_1} \left[(x - 2.5)^2 - 2.5^2 + 0.25 \right] \textcircled{M_1}$$

$$(x - 2.5)^2 - 6 \textcircled{A_1}$$

.....
(3 marks)

Express $x^2 + 6\sqrt{2}x - 1$ in the form $(x + a)^2 + b$

$$\left[(x + 3\sqrt{2})^2 - (3\sqrt{2})^2 - 1 \right] \textcircled{M_1}$$

$$(x + 3\sqrt{2})^2 - 19$$

$$\textcircled{A_1}$$

.....
(3 marks)

$$x^2 + 2ax + b \equiv (x - 5)^2 - a$$

Work out the values of a and b .

m_1 $\left[\begin{array}{l} x^2 - 10x + 25 - a = b \\ -5x - 5x \end{array} \right. \therefore b = 30$

$\therefore a = -5$

B_1 A_1

$$a = \dots\dots\dots b = \dots\dots\dots$$

(3 marks)

$$(x - 8)(x + 4) = (x - a)^2 + b \text{ for all values of } x.$$

Find the value of a and the value of b .

P_1 $\left[\begin{array}{l} x^2 - 8x + 4x - 32 \\ x^2 - ax - ax + a^2 + b \\ -2ax \end{array} \right. \begin{array}{l} -4x \\ -4x = -2ax \end{array}$

$a = 2$

$4 + b = -32$

$b = -36$

P_1 A_1

$$a = \dots\dots\dots b = \dots\dots\dots$$

(3 marks)

Completing the square for an expression in the form:

ax^2+bx+c where $a \neq 1$

Q15.

AtoZrevision.com

Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$.

$$2[x^2 + 6x] + 1 \quad (M_1)$$

$$2[(x+3)^2 - 9] + 1 \quad (M_1)$$

$$2(x+3)^2 - 18 + 1$$

$$2(x+3)^2 - 17 \quad (A_1)$$

.....
(3 marks)

Express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$ where a , b and c are integers

$$3[x^2 + 4x] + 19 \quad m_1$$

$$3[(x+2)^2 - 4] + 19 \quad m_1$$

$$3(x+2)^2 - 12 + 19$$

$$3(x+2)^2 + 7 \quad A_1$$

.....
(3 marks)

Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a , b and c are integers.

$$4[x^2 - 2x] + 7 \quad (M_1)$$

$$4[(x-1)^2 - 1] + 7 \quad (M_1)$$

$$4(x-1)^2 - 4 + 7$$

$$4(x-1)^2 + 3 \quad (A_1)$$

.....
(3 marks)

$$f(x) = 3x^2 - 9x + 5$$

Given that $f(x)$ can be written in the form $a(x - b)^2 + c$, where a , b and c are constants, find the value of a , the value of b and the value of c

$$3[x^2 - 3x] + 5 \quad \text{M1}$$

$$3[(x - 1.5)^2 - 2.25] + 5 \quad \text{M1}$$

$$3(x - 1.5)^2 - \frac{27}{4} + 5$$

$$3(x - 1.5)^2 - \frac{7}{4} \quad \text{A1}$$

$$a = \dots\dots\dots b = \dots\dots\dots c = \dots\dots\dots$$

(3 marks)

Express $3x^2 - 18x + 4$ in the form $p(x + q)^2 + r$.

$$3[x - 6x] + 4 \quad M_1$$

$$3[(x - 3)^2 - 9] + 4 \quad M_1$$

$$3(x - 3)^2 - 27 + 4$$

$$3(x - 3)^2 - 23 \quad A_1$$

.....
(4 marks)

$$4x^2 + 8x + 3 \equiv a(x + b)^2 + c.$$

Find the values of the constants a , b and c .

$$4 \left[x^2 + 2x \right] + 3 \quad m_1$$

$$4 \left[(x+1)^2 - 1 \right] + 3 \quad m_1$$

$$4(x+1)^2 - 4 + 3$$

$$4(x+1)^2 - 1 \quad A_1$$

$$a = \dots\dots\dots b = \dots\dots\dots c = \dots\dots\dots$$

(4 marks)

Express $-3x^2 - 6x + 7$ in the form $a(x + b)^2 + c$.

$$-3[x^2 + 2x] + 7 \quad \text{M1}$$

$$-3[(x+1)^2 - 1] + 7 \quad \text{M1}$$

$$-3(x+1)^2 + 3 + 7$$

$$-3(x+1)^2 + 10 \quad \text{A1}$$

.....
(3 marks)

Express $-6x^2 + 24x - 25$ in the form $p(x + q)^2 + r$.

$$-6[x - 4x] - 25 \quad \text{M1}$$

$$-6[(x-2)^2 - 4] - 25 \quad \text{M1}$$

$$-6(x-2)^2 + 24 - 25$$

$$-6(x-2)^2 - 1 \quad \text{A1}$$

.....
(3 marks)

Express $-2x^2 - 16x - 9$ in the form $a(x + b)^2 + c$, where a , b and c are integers.

$$-2[x^2 + 8x] - 9 \quad \text{M1}$$

$$-2[(x+4)^2 - 16] - 9 \quad \text{M1}$$

$$-2(x+4)^2 + 32 - 9$$

$$-2(x+4)^2 + 23 \quad \text{A1}$$

.....
(3 marks)

Express $4 + 12x - 2x^2$ in the form $a(x + b)^2 + c$.

$$-2x^2 + 12x + 4$$

$$-2[x^2 - 6x] + 4 \quad \text{M1}$$

$$-2[(x - 3)^2 - 9] + 4 \quad \text{M1}$$

$$-2(x - 3)^2 + 18 + 4 \quad \text{M1}$$

$$-2(x - 3)^2 + 22 \quad \text{A1}$$

.....
(4 marks)

Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a , b and c are integers.

$$-3x^2 + 12x + 7$$

$$-3[x^2 - 4x] + 7 \quad m_1$$

$$-3[(x-2)^2 - 4] + 7 \quad m_1$$

$$-3(x-2)^2 + 12 + 7 \quad m_1$$

$$-3(x-2)^2 + 19 \quad A_1$$

.....
(4 marks)

$$f(x) = 2 + \frac{4}{5}x - \frac{1}{25}x^2$$

Given that $f(x)$ can be expressed in the form $A - B(x + C)^2$ where A , B and C are constants, find the value of A , the value of B and the value of C .

$$2 - \frac{1}{25}(x^2 - 20x) \quad (M_1)$$

$$2 - \frac{1}{25}[(x - 10)^2 - 100] \quad (M_1)$$

$$6 - \frac{1}{25}(x - 10)^2 \quad (A_1)$$

(M_1)

$$A = \dots\dots\dots B = \dots\dots\dots C = \dots\dots\dots$$

(4 marks)

Given that $5x^2 + px - 8 = q(x - 1)^2 + r$ for all values of x , find the values of the constants p , q and r .

$$5(x-1)^2 + r$$

B_1

$$5(x^2 - 2x + 1) + r$$

B_1

$$5x^2 - 10x + 5 + r$$

M_1

$$p = -10$$

$$r = -13$$

A_1

$p = \dots\dots\dots q = \dots\dots\dots r = \dots\dots\dots$

(4 marks)