# SURDS: PROBLEM SOLVING

GRADES 8-9

AtoZrevision

## Examination practice questions

**You should have:**

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

### Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

### Information

- The marks for each question are shown in brackets.
- If the number of marks for two similar questions isn't the same, this is likely due to them being modelled on different specifications. In this case, it is worth considering both mark schemes.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions will generally get more challenging as the document progresses. Some of the latter questions will go beyond the core grade level for this topic.

### Advice

- Read each question carefully before you start to answer it.
- Don't forget to have fun.
- Check your answers at the end.

## Q1. 

Here is a sequence.

$2 \quad 2\sqrt{7} \quad 14 \quad 14\sqrt{7}$

The $n$th term of this sequence is $2 \times (\sqrt{7})^{n-1}$

Find the value of the 21st term divided by the 17th term.

$$\frac{2 \times (7^{\frac{1}{2}})^{20}}{2 \times (7^{\frac{1}{2}})^{16}} = \frac{\cancel{2} \times 7^{\cancel{10}\,2}}{\cancel{2} \times 7^{\cancel{8}}}$$

M1 M1

$7^2 = 49$

A1

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**(3 marks)**

## Q2. 

Given that $a$ is a positive integer, show that $\sqrt{3a}(\sqrt{12a} + a\sqrt{3a})$ is always a multiple of 3.

$\sqrt{36a^2} + a\sqrt{9a^2}$

$= 6a + (a \times 3a)$

M1

$6a + 3a^2$ A1

$3(2a + a^2)$

MULTIPLE OF 3

A1

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**(3 marks)**

**Q3.** AtoZrevision.com

Given that $(5-\sqrt{x})^2 = y - 20\sqrt{2}$ where $x$ and $y$ are positive integers, find the value of $x$ and the value of $y$.

$(5-\sqrt{x})(5-\sqrt{x}) =$

$5^2 - 5\sqrt{x} - 5\sqrt{x} + x$ M1

$25 + x - 10\sqrt{x} = y - 20\sqrt{2}$

$10\sqrt{x} = 20\sqrt{2}$

$\therefore x = 8$

$25 + 8 = y = 33$

A1 A1

$x$ = 8 $y$ = 33

**(3 marks)**

**Q4.** AtoZrevision.com

Given that $x$ and $y$ are positive integers such that $(1+\sqrt{x})(3+\sqrt{x}) = y + 4\sqrt{5}$ find the value of $x$ and the value of $y$.

$3 + \sqrt{x} + 3\sqrt{x} + x$ M1

$3 + x + 4\sqrt{x} = y + 4\sqrt{5}$

$x = 5$

$3 + 5 = y = 8$

A1 A1

$x$ = 5 $y$ = 8

**(3 marks)**

**Q5.** AtoZrevision.com

$$\left(\sqrt{a}+\sqrt{8a}\right)^2 = 54 + b\sqrt{2}$$

$a$ and $b$ are positive integers.
Find the value of $a$ and the value of $b$.

$a + 8a + 2\sqrt{a}\sqrt{8a}$ M1

$= 9a + 2a\sqrt{8}$

$\sqrt{8} = 2\sqrt{2}$

$\therefore 9a + 4a\sqrt{2}$ A1

$54 + b\sqrt{2}$

$\therefore a = 6$ $\therefore b = 24$ A1

$a =$ .................... $b =$ ..........................

**(3 marks)**

**Q6.** AtoZrevision.com

Show that $(4+3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$. where $k$ is a constant to be found.

$(4+3\sqrt{x})(4+3\sqrt{x})$ M1

$16 + 12\sqrt{x} + 12\sqrt{x} + 9x$ M1

$24\sqrt{x}$

$\therefore k = 24$ A1

$k =$ ..........................

**(3 marks)**

Solve the equation

$$\times\sqrt{2} \quad 10 + x\sqrt{8} = \frac{6x}{\sqrt{2}} \quad \times\sqrt{2}$$

M1

Give your answer in the form $a\sqrt{b}$ where $a$ and $b$ are integers.

$10\sqrt{2} + \underline{x\sqrt{16}} = 6x$ A1

$10\sqrt{2} + \underline{4x} = 6x$

M1

$10\sqrt{2} = 2x$

$5\sqrt{2} = x$ A1

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**(4 marks)**

The area of a rectangle is 18 cm $^2$

The length of the rectangle is $(\sqrt{7}+1)$ cm.

$l \times w = 18$

Without using a calculator, find the width of the rectangle.

Give your answer in the form $a\sqrt{b}+c$ where $a$, $b$ and $c$ are integers.

$$\frac{18}{\sqrt{7}+1} = w$$

$$\frac{18}{\sqrt{7}+1} \times \underbrace{\frac{\sqrt{7}-1}{\sqrt{7}-1}}_{M1} = \frac{18\sqrt{7}-18}{\cancel{7-1}\ 6} \quad M1$$

$\div 6$

$$= 3\sqrt{7}-3 \quad A1$$

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**(3 marks)**

A rectangle $R$ has a length $(1+\sqrt{5})$ cm and an area of $\sqrt{80}$ cm $^2$.

Calculate the width of $R$ in cm. Express your answer in the form $p+q\sqrt{5}$, where $p$ and $q$ are integers to be found.

$$l \times w = \sqrt{80}$$

$$\frac{\sqrt{80}}{\sqrt{5}+1} = w \quad B1$$

$$\frac{\sqrt{80}}{\sqrt{5}+1} \times \underbrace{\frac{\sqrt{5}-1}{\sqrt{5}-1}}_{M1} = \frac{20-4\sqrt{5}}{\cancel{5-1}\ 4} \quad M1$$

$\div 4$

$$5-\sqrt{5} \quad A1$$

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**(4 marks)**

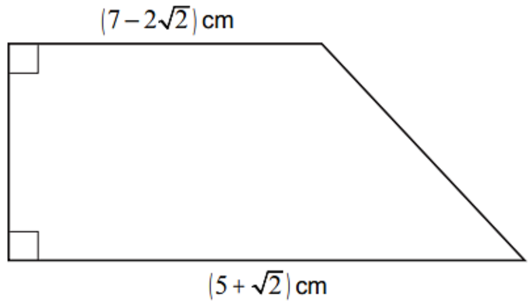


*Diagram not drawn to scale*

The area of this trapezium is $(6\sqrt{2} - 1)$ cm $^2$.

Find the height of the trapezium.
Give your answer in its simplest form.

$$\tfrac{1}{2}\left(7 - 2\sqrt{2} + 5 + \sqrt{2}\right) \times h = 6\sqrt{2} - 1$$

B1

$$\therefore \quad h = \frac{2(6\sqrt{2}-1)}{12-\sqrt{2}}$$

M1

$$\frac{12\sqrt{2}-2}{12-\sqrt{2}} \times \frac{12+\sqrt{2}}{12+\sqrt{2}}$$

M1

$$= \frac{\cancel{144\sqrt{2}} + \cancel{24} - \cancel{24} - \cancel{2\sqrt{2}}}{144-2}$$

M1

$$= \frac{142\sqrt{2}}{142} = \sqrt{2}$$

A1

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**(5 marks)**

A trapezium *ABCD* has an area of $5\sqrt{6}$ cm².

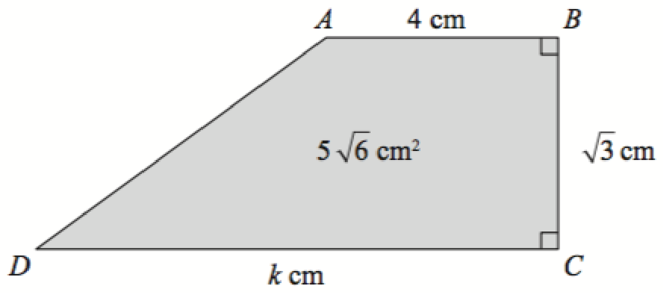


Diagram **NOT** accurately drawn

$AB = 4$ cm.
$BC = \sqrt{3}$ cm.
$DC = k$ cm.

Calculate the value of $k$, giving your answer in the form $a\sqrt{b} - c$, where $a$, $b$ and $c$ are positive integers.

$$\frac{1}{2}(4+k) \times \sqrt{3} = 5\sqrt{6}$$ M1

$$(k+4) \times \sqrt{3} = 10\sqrt{6}$$

$$k + 4 = \frac{10\sqrt{6}}{\sqrt{3}}$$

$$k = \frac{10\sqrt{6}}{\sqrt{3}} - 4$$ M1

$$= 10\sqrt{2} - 4$$ A1

$$\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$

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**(3 marks)**

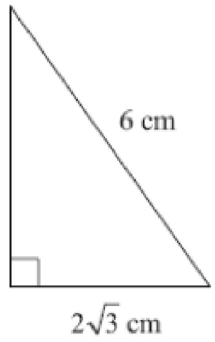


Diagram NOT accurately drawn

The diagram shows a right-angled triangle.

The length of the base of the triangle is $2\sqrt{3}$ cm.

The length of the hypotenuse of the triangle is 6 cm. The area of the triangle is $A$ cm$^2$

Show that $A = k\sqrt{2}$ giving the value of $k$.

M1

$6^2 - (2\sqrt{3})^2 = 36 - 12 = 24$

M1

A1

$A = \frac{1}{2} \times 2\sqrt{3} \times \sqrt{24} = \sqrt{72}$

A1

$\sqrt{72} = 6\sqrt{2}$

A1

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**(5 marks)**

$a = \sqrt{8} + 2$

$b = \sqrt{8} - 2$

$T = a^2 - b^2$

Work out the value of $T$.
Give your answer in the form $c\sqrt{2}$ where $c$ is an integer.

$(\sqrt{8} + 2)(\sqrt{8} + 2) =$

$8 + 4\sqrt{8} + 4$ M1

$(\sqrt{8} - 2)(\sqrt{8} - 2) =$

$8 - 4\sqrt{8} + 4$ M1

$(\cancel{8} + 4\sqrt{8} + \cancel{4}) - (\cancel{8} - 4\sqrt{8} \cancel{+4})$ M1

$8\sqrt{8} = 16\sqrt{2}$ A1

$\sqrt{8} = 2\sqrt{2}$

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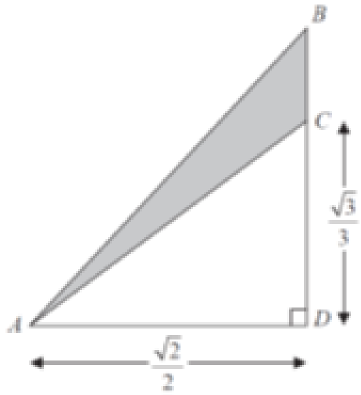
**(4 marks)**

$ABD$ is a right-angled triangle.




All measurements are given in centimetres.

$C$ is the point on $BD$ such that $CD = \frac{\sqrt{3}}{3}$

$AD = BD = \frac{\sqrt{2}}{2}$

Work out the exact area, in cm$^2$, of the shaded region.

M1 M1

$$\frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}\right)$$

$$= \frac{1}{4} - \frac{\sqrt{6}}{12}$$

A1

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**(3 marks)**

**Q15.** 

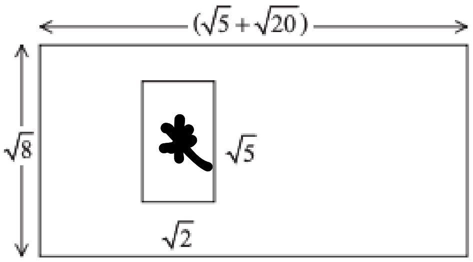


Diagram **NOT** accurately drawn

A large rectangular piece of card is $(\sqrt{5}+\sqrt{20})$ cm long and $\sqrt{8}$ cm wide.
A small rectangle $\sqrt{2}$ cm long and $\sqrt{5}$ cm wide is cut out of the piece of card.

Express the area of the card that is left as a percentage of the area of the large rectangle.

$$* = \sqrt{8}(\sqrt{5}+\sqrt{20}) - (\sqrt{2}\times\sqrt{5})$$

B1

$$= 6\sqrt{10} - \sqrt{10}$$

M1

$$\frac{6\sqrt{10}-\sqrt{10}}{6\sqrt{10}} \times 100 = \frac{500}{6}$$

M1 A1

$$\sqrt{8}(\sqrt{5}+\sqrt{20})$$

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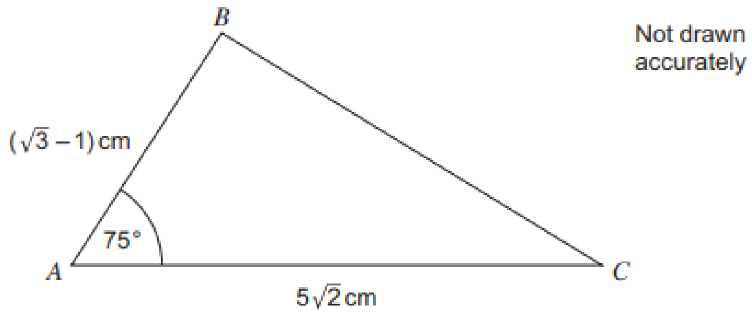
**(4 marks)**




You are given that $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\frac{1}{2}ab\sin C$ B1

Find the area of triangle $ABC$

$$\frac{1}{2} \times 5\sqrt{2} \times (\sqrt{3}-1) \times \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$(\sqrt{3}-1)(\sqrt{3}+1) = 3-1 = 2$$ M1

$$\frac{1}{2} \times \frac{5\sqrt{2}}{1} \times \frac{2}{2\sqrt{2}}$$

$$= \frac{10\sqrt{2}}{4\sqrt{2}} = \frac{5}{2}$$ A1

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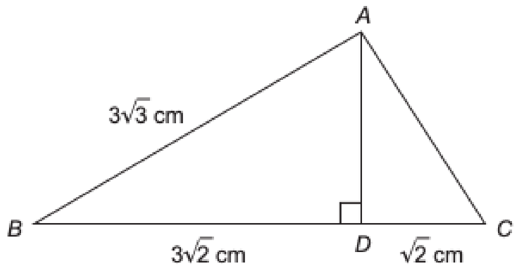
**(3 marks)**

**Q17.** 

$ABC$ is a triangle.
$AD$ is perpendicular to $BC$.
$AB = 3\sqrt{3}$ cm, $BD = 3\sqrt{2}$ cm, $DC = \sqrt{2}$ cm




Not drawn accurately

Work out the area of triangle $ABC$.
Give your answer in the form $a\sqrt{2}$ where $a$ is an integer.

$(3\sqrt{3})^2 - (3\sqrt{2})^2 = 27 - 18$
$= 9$ M1

$\sqrt{9} = \underline{\underline{3}}$ A1

$3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$ M1

$\underbrace{\tfrac{1}{2} \times 3 \times 4\sqrt{2}}_{\text{M1}} = 6\sqrt{2}$ A1

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**(5 marks)**

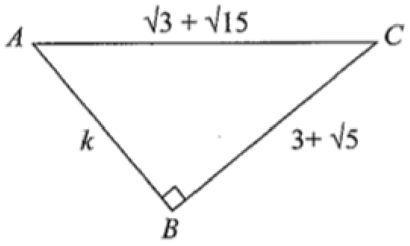


Diagram **NOT** accurately drawn

All measurements on the triangle are in centimetres.

$ABC$ is a right-angled triangle.
$k$ is a positive integer.

Find the value of $k$.

$$(3+\sqrt{5})^2 + k^2 = (\sqrt{3}+\sqrt{15})^2$$

M1

$$\cancel{9} + \cancel{6\sqrt{5}} + \cancel{5} + k^2 = \cancel{3} + \cancel{2\sqrt{45}} + \cancel{15}$$

M1

$$-4 + k^2 = 0$$

$$k^2 = 4$$

$$k = 2$$

A1

$\sqrt{45} = \sqrt{9} \times \sqrt{5}$

$= 3\sqrt{5}$

$\therefore 2\sqrt{45} = 6\sqrt{5}$

B1

CANCELLING $\sqrt{2}$ TERMS

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**(4 marks)**

What is the median of the following numbers?

$(9\sqrt{2})^2$ $(3\sqrt{19})^2$ $(4\sqrt{11})^2$ $(5\sqrt{7})^2$ $(6\sqrt{5})^2$

M1

162 171 176 175 180 A1

$\cancel{9}\sqrt{2}$ $\cancel{3}\sqrt{19}$ $5\sqrt{7}$ $\cancel{4}\sqrt{11}$ $\cancel{6}\sqrt{5}$

A1

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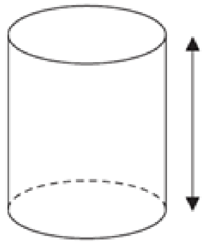
**(3 marks)**

The diagram shows a solid cylinder.




The cylinder has radius $4\sqrt{3}$ cm and height $h$ cm.
The total surface area of the cylinder is $56\pi\sqrt{6}\ cm^2$.

Find the exact value of $h$.
Give your answer in the form $a\sqrt{2} + b\sqrt{3}$ where $a$ and $b$ are integers.

$$2\pi \times (4\sqrt{3})^2 + 2\pi \times 4\sqrt{3}h = 56\pi\sqrt{6}$$ M1

$$96\pi + 8\pi \times \sqrt{3}h = 56\pi\sqrt{6}$$ M1

$$h = \frac{56\pi\sqrt{6} - 2\pi(4\sqrt{3})^2}{2\pi(4\sqrt{3})}$$ M1

$$= \frac{56\sqrt{6} - 96}{8\sqrt{3}} = 7\sqrt{2} - 4\sqrt{3}$$ A1

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**(4 marks)**

## Q21. AtoZrevision.com

Find the value of the expression

$$\sqrt{8-4\sqrt{2}+1}+\sqrt{9-12\sqrt{2}+8}$$

$\sqrt{(2\sqrt{2}-1)^2}$ $\sqrt{(3-2\sqrt{2})^2}$

$= 2\sqrt{2}-1$ $= 3-2\sqrt{2}$

$2\sqrt{2}-1+3-2\sqrt{2} = 2$

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**(3 marks)**

## Q22. AtoZrevision.com

Circle the smallest value.

| $10-3\sqrt{11}$ | $8-3\sqrt{7}$ | $5-2\sqrt{6}$ | $9-4\sqrt{5}$ | $7-4\sqrt{3}$ |
|---|---|---|---|---|
| $\sqrt{100}-\sqrt{99}$ | $\sqrt{64}-\sqrt{63}$ | $\sqrt{25}-\sqrt{24}$ | $\sqrt{81}-\sqrt{80}$ | $\sqrt{49}-\sqrt{48}$ |

SMALLEST

AS INCREASING THE VALUES DECREASES OVERALL RESULT

FANCY VERSION: AS N INCREASES, $\sqrt{n+1}-\sqrt{n}$ DECREASES.

**(3 marks)**

The $n$th term in a certain sequence is calculated by multiplying together all the numbers $\sqrt{1+\frac{1}{k}}$ where $k$ takes all the integer values from 2 to $n+1$ inclusive. For example, the third term in the sequence is $\sqrt{1+\frac{1}{2}} \times \sqrt{1+\frac{1}{3}} \times \sqrt{1+\frac{1}{4}}$.

Which is the smallest value of $n$ for which the $n$th term of the sequence is an integer?

$= \sqrt{\frac{3}{2}} \times \sqrt{\frac{4}{3}} \times \sqrt{\frac{5}{4}} \times \sqrt{\frac{6}{5}} \ldots\ldots$

EACH NUMERATOR IS CANCELLED BY THE FOLLOWING DENOMINATOR.

LEAVING 1ST NUMERATOR AND FINAL DENOMINATOR.

$\therefore$ NTH TERM $= \frac{\sqrt{n+2}}{\sqrt{2}} = \sqrt{\frac{n+2}{2}}$

AS $N \geq 1$, PRODUCT $\neq 1$

FOR 2: $\sqrt{\frac{6+2}{2}} = 2$

WHICH WE GET IF $\underline{\underline{N=6}}$

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**(3 marks)**

PHEW!

## Q24. 

For which value of $k$ is $\sqrt{2016} + \sqrt{56}$ equal to $14^k$?

$\sqrt{14} \times \sqrt{4}$

$= 2\sqrt{14}$

$\sqrt{144} \times \sqrt{14}$

$12\sqrt{14}$

$= 14\sqrt{14}$

$= 14^1 \times 14^{1/2} = 14^{3/2}$ !

(3 marks)

## Q25. 

Two squares, each of side length $1 + \sqrt{2}$ units, overlap. The overlapping region is a regular octagon.

What is the area of the octagon? Give your answer in square units.

$2 + 2\sqrt{2}$

(3 marks)