HARDER INDICES

GRADE 9

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

GRADE 9 PROBLEMS ARE IN GREEN. THEY ARE PRECEEDED BY GRADE 7/8 LEVEL EXAM QUESTIONS OF A SIMILAR NATURE IN ORDER TO HELP BUILD YOUR UNDERSTANDING BEFORE ATTEMPTING THEM. WHILST ALL OF THE QUESTIONS ARE FROM PAST PAPERS, YOU MAY WANT TO ONLY ATTEMPT THE PROBLEMS IN GREEN IF YOU FEEL THAT YOUR UNDERSTANDING IS VERY STRONG.

Q1. AtoZrevision.com

Find the value of $9^{-\frac{1}{2}}$

$9^{\frac{1}{2}} = 3$ M1

$3^{-1} = \frac{1}{3}$ A1

.....

(2 marks)

Q2. AtoZrevision.com

Carol says that $64^{-\frac{1}{2}} = \frac{1}{32}$.

She has made an error. Give the correct value of $64^{-\frac{1}{2}}$ in the form $\frac{p}{q}$.

$64^{\frac{1}{2}} = 8$ M1

A1

$8^{-1} = \frac{1}{8}$

.....

(2 marks)

Q3. AtoZrevision.com

Simplify completely $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

$\frac{(16w^8)^{3/4}}{(y^{20})^{3/4}} = \left(\frac{8w^6}{y^{15}}\right)^{-1}$ M1

$= \frac{y^{15}}{8w^6}$ A1

.....

(2 marks)

Q4. AtoZrevision.com

Simplify fully

$$\left(\frac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$$

$$\frac{(256x^{20})^{1/4}}{(y^8)^{1/4}} = \left(\frac{64x^5}{y^2}\right)^{-1} = \frac{y^2}{64x^5}$$

M1 A1

.....

(2 marks)

Q5. AtoZrevision.com

Simplify fully

$$\left(\frac{125e^{12}}{27f^3}\right)^{-\frac{2}{3}}$$

$$\frac{(125e^{12})^{2/3}}{(27f^3)^{2/3}} = \left(\frac{25e^8}{9f^2}\right)^{-1} = \frac{9f^2}{25e^8}$$

M1 A1

.....

(3 marks)

Q6. AtoZrevision.com

Work out the value of

$$\frac{\left(5\frac{4}{9}\right)^{-\frac{1}{2}} \times \left(4\frac{2}{3}\right)}{2^{-3}}$$

You must show all your working.

$\left(\frac{49}{9}\right)^{-\frac{1}{2}} \times \frac{14}{3} = \frac{3}{7} \times \frac{14}{3} = \frac{42}{21}$ (M1) (B1) (A1)

$\frac{42}{21} \div \frac{1}{8} = \frac{42}{21} \times \frac{8}{1} = \frac{336}{21} = 16$ (M1)

.....

(4 marks)

Q7. AtoZrevision.com

$$\frac{18 \times \left(\sqrt{27}\right)^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n

Show your working clearly and simplify your expression.

$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$ (M1)

$\sqrt{27} = (3^3)^{\frac{1}{2}} = 3^{3/2} \rightarrow \left(3^{\frac{3}{2}}\right)^{4n+6} = 3^{6n+9}$ (M1)

$\frac{3 \times 3^{6n+9}}{6 \times 3^{4n+16}} = \frac{3^1 \times 3^{6n+9}}{3^{4n+16}}$ (M1)

$= \frac{3^{6n+10}}{3^{4n+16}} = 3^{2n-6}$ (A1)

.....

(3 marks)

Find the values of *n* such that

$$\frac{10^{4n} \times 2^{3(n^2-5n)} \times 5^{2(1-2n)}}{20^2} = 1$$

Show clear algebraic working.

$10^{4n} = (5 \times 2)^{4n} = 5^{4n} \times 2^{4n}$

$$\frac{5^{4n} \times 2^{4n} \times 2^{3n^2-5n} \times 5^2}{5^{4n} \times (5 \times 2^2)^2} \quad M_1$$

$(5 \times 2^2)^2$ M_1

$$\frac{5^{4n} \times 2^{3n^2-15n} \times 5^2}{5^{4n} \times 5^2 \times 2^4} = 1$$

$2^{3n^2-15n} = 2^4$ M_1 A_1

$3n^2 - 15n = 4$

$3n^2 - 15n - 4 = 0$ M_1

$(3n+1)(n-4)$

$n = -\frac{1}{3}$

$n = 4$

(5 marks)

Q9. AtoZrevision.com

Write $\sqrt[3]{2}$ as a power of 2.

$2^{1/3}$

.....

(1 mark)

Q10. AtoZrevision.com

$8\sqrt{8}$ can be written in the form 8^k

Find the value of k.

$8 \times 8^{1/2}$ $8^{3/2}$

.....

(1 mark)

Q11. AtoZrevision.com

$$\frac{\sqrt{a} \times a}{a^{-2}}$$

can be written in the form a^k

Find the value of k.

$\frac{a^{1/2} \times a^1}{a^{-2}} = \frac{a^{3/2}}{a^{-2}} = a$ A1 3.5

M1

.....

(2 marks)

Q12. AtoZrevision.com

Express

$$\frac{1}{2\sqrt{2}}$$

as a power of 2.

$\frac{1}{2^1 \times 2^{1/2}} = \frac{1}{2^{3/2}}$ A1

M1

$2^{-3/2}$

.....

(2 marks)

Q13. AtoZrevision.com

$5\sqrt{5}$ can be written in the form 5^k.

Find the value of k.

$5^1 \times 5^{1/2} = 5^{3/2}$

$5^{3/2}$

.....

(1 mark)

Q14. AtoZrevision.com

Write $\sqrt{\frac{1}{m^6}}$ as a single power of m.

$\left(\frac{1}{m^6}\right)^{1/2} = \frac{1^{1/2}}{(m^6)^{1/2}} = \frac{1}{m^3}$ A1

M1

.....

(2 marks)

Q15. AtoZrevision.com

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m.

$\frac{1}{p^1 \times p^{2/3}} = \frac{1}{p^{5/3}}$ M1

$p^{-5/3}$ A1

.....

(2 marks)

Q16. AtoZrevision.com

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = a^k$$

Work out the value of k.

$\frac{a^{3/2}}{a^{2/3}} = \frac{a^{9/6}}{a^{4/6}} = a^{5/6}$

M1 m1 A1

.....

(3 marks)

Q17. AtoZrevision.com

$\sqrt[3]{a^4} \times \frac{1}{a}$ can be written in the form a^n.

State the value of n.

$a^{4/3} \times a^{-1} = a^{-4/3}$

B1 B1 A1

$-4/3$

(3 marks)

Q18. AtoZrevision.com

Given that $n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$ where $n > 0$

find the value of n.

M1 $n^{-4/5} = \frac{1}{16}$

M1 $n^{-4} = \left(\frac{1}{16}\right)^5$

M1 $n = \left(\frac{1}{16}\right)^{-5/4} = 32$ A1

(4 marks)

Q19. AtoZrevision.com

$\left(\sqrt{\frac{y}{x}}\right)^{-5} = \frac{x^m}{y^m}$

where $x \neq y$, find the value of m.

$\left(\frac{y^{1/2}}{x^{1/2}}\right)^{-5} = \left(\frac{y^{5/2}}{x^{5/2}}\right)^{-1} = \frac{x^{5/2}}{y^{5/2}}$

M1 M1 A1

m

(3 marks)

Q20. AtoZrevision.com

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m.

SEE Q15 :o

.....

(3 marks)

Q21. AtoZrevision.com

Given that $a = 3^x$ and $b = 3^y$ express in terms of a or b or a and b,

3^{x+4y}

$3^x \times 3^y \times 3^y \times 3^y \times 3^y$ $a \times b^4$

$a \times b \times b \times b \times b$

.....

(1 mark)

Q22. AtoZrevision.com

Given that $a = 3^x$ and $b = 3^y$ express in terms of a or b or a and b,

3^{2x}

$3^x \times 3^x = a \times a = a^2$

.....

(1 mark)

Q23. AtoZrevision.com

You are given that $x = 5^m$ and $y = 5^n$

Write 5^{3n} in terms of y.

y^3

.....

(1 mark)

Q24. AtoZrevision.com

$x = 2^p, \quad y = 2^q$

Express 2^{p-1} in terms of x and/or y.

$2^p \times 2^{-1} = \frac{x}{2}$

.....

(1 mark)

Q25. AtoZrevision.com

$x = 2^p, \quad y = 2^q$

Express 2^{2q} in terms of x and/or y.

$(2^q)^2 = (y)^2 = y^2$

.....

(1 mark)

Q26. AtoZrevision.com

$x = 2^p, \quad y = 2^q$

Express 2^{p+q} in terms of x and/or y.

$2^p \times 2^q = xy$

.....

(1 mark)

Q27. AtoZrevision.com

Given that

$$\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{8^y}$$

express n in terms of x and y.

$2^{\frac{1}{2}n} = \frac{2^x}{2^{3y}}$ M1

$2^{\frac{1}{2}n} = 2^{x-3y}$

M1

$\frac{1}{2}n = x - 3y$

$n = 2x - 6y$

.....

A1

(3 marks)

Q28. AtoZrevision.com

$x = 2^p, \quad y = 2^q$

$xy = 32$
and $2xy^2 = 32$

$\div$

Find the value of p and the value of q.

$2y = 1$ m1

$y = \frac{1}{2}$

$x = 64$

$2^p = 64$

$p = 6$

$2^q = \frac{1}{2}$

$q = -1$ A1

.....

(2 marks)

Q29. AtoZrevision.com

Solve the equation

$5^n \times 5^{n+4} = 25$

$5^{2n+4} = 5^2$ m1

$2n + 4 = 2$ m1

$\therefore n = -1$ A1

.....

(3 marks)

Q30. AtoZrevision.com

Solve the equation

$2^x \times 2^{3x-8} = 16$

$2^{4x-8} = 2^4$

$4x - 8 = 4$

$\therefore x = 3$

.....

(3 marks)

Q31. AtoZrevision.com

Solve the equation

$$5^n \times 5^{n+4} = 25$$

SEE Q29.

.....

(3 marks)

Q32. AtoZrevision.com

Solve the equation

$$(13^{-6})^4 \times 13^5 = 169^k$$

$13^{-24} \times 13^5 = 13^{2k}$ M1

$-24 + 5 = 2k$

$-19 = 2k$ M1

$k = 9.5$ A1

.....

(3 marks)

Q33. AtoZrevision.com

Find an equation for p, in terms of q, if $27 \times 3^{p+2q} = 9^{3q}$.

M1 $3^3 \times 3^{p+2q} = 3^{6q}$

M1 $3 + p + 2q = 6q$

$p = 4q - 3$ A1

.....

(3 marks)

Q34. AtoZrevision.com

$$16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$$

Work out the exact value of x.

M1

$$(2^4)^{1/5} \times 2^x = (2^3)^{3/4}$$

M1

$$2^{4/5} \times 2^x = 2^{9/4}$$

A1

$$x + 4/5 = 9/4$$

M1

$$\therefore x = 1.45$$

A1

.....

(5 marks)

Q35. AtoZrevision.com

$$(3^a)^{b+1} \times \frac{2^c}{4} = 3^{c+a} \times 2$$

Find possible values of a, b, and c.

M1

$$3^{ab} \times 3 \times 2^c \times 2^{-2} = 3^{c+a} \times 2$$

$$ab + 1 = c + a \qquad ab + 1 = 3 + a$$

$$c - 2 = 1 \qquad ab = a + 4$$

$$c = 3 \qquad \therefore a \text{ AND } b$$

A1

$$= 1 \text{ or } 3$$

A1

.....

(3 marks)

Q36. AtoZrevision.com

Solve $3 \times 4^{2k+8} = 24$
Show your working clearly.

$4^{2k+8} = 8$ M1

$(2^2)^{2k+8} = 2^3$ M1 POWERS OF 2

$2^{4k+16} = 2^3$

$4k + 16 = 3$ M1 EQUATING EXPONENTS

$4k = -13$

$k = -13/4$ A1

.....

(4 marks)

Q37. AtoZrevision.com

$100^a \times 1000^b$ can be written in the form 10^w

Express w in terms of a and b.

$(10^2)^a \times (10^3)^b$ M1

$10^{2a} \times 10^{3b} = 10^{2a+3b}$ A1 w

.....

(2 marks)

Q38. AtoZrevision.com

(a) $x = 9 \times 10^{2m}$ where m is an integer.

Find, in standard form, an expression for $\sqrt{x}$

$x^1 = 9 \times 10^{2m}$

$x^{1/2} = (9 \times 10^{2m})^{1/2} = 3 \times 10^{m}$ A1 A1

.....

(2)

(b) $y = 9 \times 10^{2n}$ where n is an integer.

Find, in standard form, an expression for $y^{\frac{3}{2}}$
Give your answer as simply as possible.

$(9 \times 10^{2n})^{3/2}$ M1

$= 27 \times 10^{3n}$ A1

NOT S.F.!

$2.7 \times 10^{3n+1}$ A1

.....

(3)

(5 marks)

Q39. AtoZrevision.com

$y = 16 \times 10^{8k}$ where k is an integer.

Find an expression, in terms of k, for $y^{\frac{5}{4}}$
Give your answer in standard form.

$(16 \times 10^{8k})^{5/4}$ M1

32×10^{10k} A1

$3.2 \times 10^{10k+1}$ A1

.....

(3 marks)

Q40. AtoZrevision.com

$x = a \times 10^n$ where n is an integer and $\sqrt{10} \leqslant a < 10$

Find, in standard form, an expression for x^2.
Give your expression as simply as possible.

$(a \times 10^n)^2$ M1

$= a^2 \times 10^{2n}$ A1

$\therefore a^2$ WILL BE 10 OR MORE $\therefore \div 10$ FOR S.F.

$\frac{a^2}{10} \times 10^{2n+1}$ A1

.....

(3 marks)

Q41. AtoZrevision.com

$m = 8 \times 10^{9n}$ where n is an integer.

Express $m^{-\frac{1}{3}}$ in standard form.
Give your answer, in terms of n, as simply as possible.

$(8 \times 10^{9n})^{-1/3}$ M1

$2^{-1} \times 10^{-3n}$

0.5×10^{-3n} A1

NOT S.F.

$5 \times 10^{-3n-1}$ A1

.....

(3 marks)

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

Given that $4^x+4^x+4^x+4^x = 4^{16}$, what is the value of x?

$4^x(4) = 4^{16}$

$4^x \times 4^1 = 4^{16}$

$x + 1 = 16$

$\therefore x = 15$ ☺

Q2. AtoZrevision.com

Suppose that $8^m = 27$.

What is the value of 4^m?

$(2^3)^m = 2^{3m} = (2^m)^3 = (3)^3$

$2^m = 3$

$4^m = (2^2)^m$

$= 2^m \times 2^m = 3 \times 3 = 9.$

You are given that $5^p = 9, 9^q = 12, 12^r = 16, 16^s = 20$ and $20^t = 25$.

What is the value of $pqrst$?

$(5^p)^q = 12$

$5^{pq} = 12$

$(5^{pq})^r = 16$

$5^{pqr} = 16$

$(5^{pqr})^s = 20$

$5^{pqrs} = 20$

$(5^{pqrs})^t = 25$

$5^{pqrst} = 5^2$

$pqrst = 2$

PHEW!