INVERSE FUNCTIONS

GRADE 8

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

Q1. AtoZrevision.com

The functions f and g are such that
$f(x) = 3x - 1$ and
$g(x) = x^2 + 4$

Find $f^{-1}(x)$

$y = 3x - 1$

$y + 1 = 3x$ M1

$\frac{y+1}{3} = x$ M1

A1

$f^{-1}(x) = \frac{x+1}{3}$

(2 marks)

Q2. AtoZrevision.com

The functions f and g are defined as

$f(x) = \frac{1}{2}x + 4$

$g(x) = \frac{2x}{x+1}$

Express the inverse function f^{-1} in the form $f^{-1}(x)$

$y = \frac{1}{2}x + 4$ M1

$y - 4 = \frac{1}{2}x$

$2(y - 4) = x$ M1

$f^{-1}(x) = 2(x - 4)$

(3 marks)

A1

Q3.	AtoZrevision.com

g is the function with domain $x \geq -3$ such that $g(x) = x^2 + 6x$

Express the inverse function g^{-1} in the form $g^{-1}(x)$

$$y = x^2 + 6x$$

$$y = (x+3)^2 - 9$$ M1

$$y + 9 = (x+3)^2$$

M1

$$\sqrt{y+9} = x + 3$$

$$\sqrt{y+9} - 3 = x$$

A1

$f^{-1}(x) = \sqrt{x+9} + 3$

(3 marks)

Q4.	AtoZrevision.com

$$f(x) = 2x - 3$$
$$g(x) = \frac{x}{3x+1}$$

Express the inverse function g^{-1} in the form $g^{-1}(x)$

$$y = \frac{x}{3x+1}$$ M1

$$3xy + y = x$$

A1

M1 $$3xy - x = -y$$

$$x(3y-1) = -y$$

$$-x = \frac{y}{3y-1}$$

$f^{-1}(x) = \frac{x}{1-3x}$

(3 marks)

Q5.

The functions f and g are defined as

$f(x) = \frac{x}{4x-3}$ and $g(x) = x - 5$

Express the inverse function f^{-1} in the form $f^{-1}(x)$

$$y = \frac{x}{4x-3}$$

M1 $4xy - 3y = x$

M1 $4xy - x = 3y$

$x(4y-1) = 3y$

$$x = \frac{3y}{4y-1}$$

A1

$$f^{-1}(x) = \frac{3x}{4x-1}$$

(3 marks)

Q6.

$f: x \to 2x^2 + 1 \quad g: x \to \frac{2x}{x-1}$ where $x \neq 1$

Express the inverse function g^{-1} in the form $g^{-1}: x \to$

$$y = \frac{2x}{x-1}$$

$xy - y = 2x$ M1

$xy - 2x = y$

$x(y-2) = y$ M1

$$x = \frac{y}{y-2}$$

A1

$$f^{-1}(x) = \frac{x}{x-2}$$

(3 marks)

Q7.	**AtoZrevision.com**

The function g is such that $g(x) = 2x^2 - 20x + 9$ where $x \geq 5$.

Express the inverse function g^{-1} in the form $g^{-1}(x)$

$y - 9 = 2x^2 - 20x$

$y - 9 = 2\left[(x-5)^2 - 25\right]$ M1

$y - 9 = 2(x-5)^2 - 50$ M1

$y + 41 = 2(x-5)^2$

$\sqrt{\dfrac{y+41}{2}} = x - 5$ M1

$g^{-1}(x) = 5 + \sqrt{\dfrac{x+41}{2}}$ A1

(4 marks)

Q8.	**AtoZrevision.com**

For all values of x

$f(x) = (x+1)^2$ and $g(x) = 2(x-1)$

Find $g^{-1}(7)$.

$y = 2x - 2$

$y + 2 = 2x$

$\dfrac{y+2}{2} = x$ M1

$\dfrac{7+2}{2} = x$

$g^{-1}(7) = 4.5$ A1

(2 marks)

Q9.	**AtoZrevision.com**

f and g are functions such that

$f(x) = \frac{12}{\sqrt{x}}$ and $g(x) = 3(2x+1) = 6x + 3$

Find $g^{-1}(6)$

$y - 3 = 6x$

$\dfrac{y-3}{6} = x = g^{-1}(x)$ M1

$\dfrac{6-3}{6} = \dfrac{1}{2} = x$

$g^{-1}(6) = \dfrac{1}{2}$ A1

(2 marks)

Q10. AtoZrevision.com

The function f is given by $f(x) = 2x^3 - 4$

Find the value of $f^{-1}(50)$

$y = 2x^3 - 4$

$y + 4 = 2x^3$

$\frac{y+4}{2} = x^3$ M1

$\sqrt[3]{\frac{y+4}{2}} = x = g^{-1}(x)$

$\sqrt[3]{\frac{50+4}{2}} = g^{-1}(x)$

$= 3$ A1

$f^{-1}(50) = 3$ ....

(2 marks)

Q11. AtoZrevision.com

The functions g and h are such that

$y = \cancel{g(x)} = \sqrt[3]{2x - 5}$ $\quad y = \cancel{h(x)} = \frac{1}{x}$

$\frac{1}{y} = x = h^{-1}(x)$ M1

Find $hg^{-1}(x)$

Give your answer in terms of x in its simplest form.

$y^3 = 2x - 5$

$y^3 + 5 = 2x$ M1

$\frac{y^3+5}{2} = x = g^{-1}(x)$

A1

$\frac{1}{\frac{y^3+5}{2}} = \frac{2}{y^3+5}$ $\therefore hg^{-1}(x) = \frac{2}{x^3+5}$

....

(3 marks)

Q12. AtoZrevision.com

The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{where } x > 0$$

$$g(x) = \frac{4}{x^2} \quad \text{where } x > 0$$

The function h is such that $h = (fg)^{-1}$

Find $h(x)$

$$fg(x) = 3\left(\frac{4}{x^2}\right)^2 + 1$$

M1 $$= 3\left(\frac{16}{x^4}\right) + 1$$

A1 $$= \frac{48}{x^4} + 1$$

$$(fg)^{-1}$$

$$y = \frac{48}{x^4} + 1$$

$$y - 1 = \frac{48}{x^4}$$ M1

$$x^4 = \frac{48}{y-1}$$

$$\cancel{x} = \sqrt[4]{\frac{48}{y-1}}$$ A1

....

(4 marks)

Q13. AtoZrevision.com

Two functions, f and g are defined as

$$f: x \to 1 + \frac{1}{x} \qquad \text{where } x > 0$$

$$g: x \to \frac{x+1}{2} \qquad \text{where } x > 0$$

Given that $h = fg$

express the inverse function h^{-1} in the form $h^{-1}: x \to$

$$fg(x) = 1 + \frac{1}{\frac{x+1}{2}}$$ M1

$$= 1 + \frac{2}{x+1}$$

$$y = \frac{2}{x+1} + 1$$

$$y - 1 = \frac{2}{x+1}$$ M1

$$x + 1 = \frac{2}{y-1}$$ M1

$$x = \frac{2}{y-1} - 1$$

$$h^{-1} = \frac{2}{x-1} - 1$$ A1

....

(4 marks)

Q14. AtoZrevision.com

$$y \cancel{f(x)} = \frac{1-x}{1+x}$$

Find $f^{-1}(x)$.

$y(1+x) = 1-x$ M1

$y + xy = 1-x$

$xy + x = 1-y$ M1

$x(y+1) = 1-y$

$x = \frac{1-y}{y+1}$

A1

$f^{-1}(x) = \frac{1-x}{x+1}$ ....

(3 marks)

Q15. AtoZrevision.com

The function g is defined as

$g : x \to 5 + 6x - x^2$ with domain $\{x : x \geq 3\}$

Given that $g^{-1} : x \to 3 + \sqrt{14 - x}$
State the domain of g^{-1}

$x \leq 14$ ....

(1 mark)

Q16. AtoZrevision.com

The functions g and h are given as: $g(x) = x^2 - 1$ and $h(x) = e^x + 1$

Suggest a domain for g such that g^{-1} exists.

$g \geq 0$ ....

(1 mark)

The function f is such that $f(x) = 5 + 6x - x^2$ for $x \leq 3$

Find the range of values of x for which $f^{-1}(x)$ is positive.

$$y = -\left[x^2 - 6x\right] + 5$$ M1

$$y = -\left[(x-3)^2 - 9\right] + 5$$

$$y = -(x-3)^2 \cancel{+9+5} + 14$$ A1

$$(x-3)^2 = 14 - y$$ M1

$$x - 3 = \pm\sqrt{14-y}$$

$$x = \pm\sqrt{14-y} + 3$$

$$f^{-1}(x) = 3 - \sqrt{14-x}$$ M1

$$\therefore \quad 5 < x \leq 14$$ A1

....

(5 marks)

The function f is defined as

$$y \; \cancel{f(x)} = \frac{\sqrt{x^2+k^2}}{x}$$

Where $x > 0$ and where k is a positive number.

Find the value of p for which $f^{-1}(p) = k$.

$$yx = \sqrt{x^2+k^2} \quad M1$$

$$y^2x^2 = x^2+k^2$$

$$y^2x^2 - x^2 = k^2 \quad M1$$

$$x^2(y^2-1) = k^2$$

$$x^2 = \frac{k^2}{y^2-1} \quad A1$$

$$x = \frac{k}{\sqrt{y^2-1}}$$

$$k = \frac{k}{\sqrt{p^2-1}} \quad M1$$

$$\sqrt{p^2-1} = 1$$

$$M1 \quad p^2-1 = 1^2$$

$$p^2 = \cancel{1^2+1}\; 2$$

$$p = \sqrt{2} \quad A1$$

....

(6 marks)

$$f(x) = x^2$$
$$y \quad \cancel{g(x)} = x - 3$$

Solve the equation $gf(x) = g^{-1}(x)$

$y + 3 = x$

$x + 3 = g^{-1}(x)$

$gf(x) = x^2 - 3$ M1

EITHER

$x + 3 = x^2 - 3$

$0 = x^2 - x - 6$ A1

$\therefore \quad x = 3 \qquad x = -2$

A1

……………………………………….

(5 marks)

The function f and g are such that

$$f(x) = 5x + 3 \qquad g(x) = ax + b \qquad \text{where } a \text{ and } b \text{ are constants.}$$

$$g(3) = 20 \quad \text{and} \quad f^{-1}(33) = g(1)$$

$a+b$ P1

Find the value of a and the value of b.

$3a+b=20$ P1

$y=5x+3$

$\frac{y-3}{5}=x$

$\frac{x-3}{5}=f^{-1}(x)$ P1

$\therefore \frac{33-3}{5}=a+b$ P1

$30=5a+5b$

$5a+5b=30$

$\times 5$ $\cancel{3a+b=20}$

$15a+5b=100$

$-10a=-70$

$a=7$ $\therefore b=-1$ A1

PHEW!

....

(5 marks)