MULTIPLYING AND DIVIDING ALGEBRAIC FRACTIONS

GRADE 9

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

Q1.	**AtoZrevision.com**

Simplify

$$\frac{x^2-2x}{3}\times\frac{6}{x^2+2x-8}$$

$$\frac{x\cancel{(x-2)}}{3}\times\frac{6}{(x+4)\cancel{(x-2)}}$$

M1

$$\frac{\cancel{6}x}{\cancel{3}x+\cancel{12}}=\frac{2x}{x+4}$$

A1

M1

....

(3 marks)

Q2.	**AtoZrevision.com**

It can be shown that

$$\frac{2}{3-x}+\frac{3}{1+x}\equiv\frac{11-x}{(3-x)(1+x)}$$

Hence express

$$\cancel{\left(\frac{2}{3-x}+\frac{3}{1+x}\right)}\times\frac{x^2+8x-33}{121-x^2}\times\left(\frac{11-x}{(3-x)(1+x)}\right)$$

as a single fraction in its lowest terms.

$$\frac{\cancel{(x+11)}\cancel{(x-3)}}{\cancel{(11+x)}\cancel{(11-x)}}\times\frac{\cancel{11-x}}{\cancel{(3-x)}(1+x)}$$

$-1(x-3)$

M1

M1

$$\frac{-1}{1+x}$$

A1

....

(3 marks)

Simplify fully

$$\frac{n^2-1}{n+1} \times \frac{2}{n-2}$$

$$\frac{\cancel{(n+1)}(n-1)}{\cancel{(n+1)}} \times \frac{2}{(n-2)}$$

M1

M1

$$\frac{2(n-1)}{n-2}$$

A1

……………………………………

(3 marks)

Express

$$\frac{n}{n^2-4} \div \frac{3}{n-2}, \quad n \neq -2, \; n \neq 2$$

as a single fraction in its simplest form.

$$\frac{n}{(n+2)\cancel{(n-2)}} \times \frac{\cancel{n-2}}{3}$$

M1

M1

$$\frac{n}{3(n+2)}$$

A1

……………………………………

(3 marks)

Q5. AtoZrevision.com

Express

$$\frac{4}{x+2} \div \frac{5}{(x+2)^2}, \quad x \neq -2$$

as a single fraction in its simplest form.

$$\frac{4}{\cancel{x+2}} \times \frac{\cancel{(x+2)}(x+2)}{5}$$ M1

$$\frac{4(x+2)}{5}$$ A1

....

(2 marks)

Q6. AtoZrevision.com

Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

M1

$$\frac{1}{\cancel{3x-2}} \times \frac{\cancel{(3x+2)}\cancel{(3x-2)}}{\cancel{(3x+2)}(x-5)}$$ M1

$$\frac{1}{x-5} - \frac{7}{x-1}$$ M1

$$\frac{(x-1) - 7(x-5)}{(x-5)(x-1)}$$ M1

$$\frac{34-6x}{(x-5)(x-1)}$$ A1

....

(5 marks)

Express

$$\frac{25x^2-64}{5x^2-13x-6}\times\frac{x^2-8x+15}{5x+8}-(x-7)$$

as a single fraction in its simplest form.

$$\frac{(5x-8)\cancel{(5x+8)}}{(5x+2)\cancel{(x-3)}}\times\frac{(x-5)\cancel{(x-3)}}{\cancel{5x+8}}$$

M1

$$\frac{(5x-8)(x-5)}{5x+2}-(x-7)$$

M1

$$\frac{(5x-8)(x-5)-(x-7)(5x+2)}{5x+2}$$

$$\frac{5x^2-25x-8x+40-(5x^2-35x+2x-14)}{5x+2}$$

M1 FOR SIMPLIFYING

SIMPLIFIES TO:

$$\frac{54}{5x+2}$$

A1

....

(4 marks)

Express

$$\frac{4x^2-25}{5x^2+2x-7}\times\left(\frac{2}{x-3}-\frac{3}{2x-5}\right)$$

as a single fraction in its simplest form.

$$\frac{(2x+5)\cancel{(2x-5)}}{(5x+7)(x-1)}\times\frac{2(2x-5)-3(x-3)}{(x-3)(2x-5)}$$

M1

$$\frac{(2x+5)(2x-5)}{(5x+7)(x-1)}\times\frac{x-1}{(x-3)\cancel{(2x-5)}}$$

A1 M1

$$\frac{(2x+5)}{(5x+7)(x-3)}$$

A1

....

(4 marks)

It can be shown that

$$\frac{3}{2x+1} - \frac{2}{x+1} \equiv \frac{1-x}{(2x+1)(x+1)}$$

Hence express

$$\left(\frac{3}{2x+1} - \frac{2}{x+1}\right)\left(\frac{6x+3}{x^2+x-2}\right)$$

as a single algebraic fraction in its lowest terms.

$$\underbrace{\frac{\cancel{1-x}}{\cancel{(2x+1)}(x+1)} \times \frac{3\cancel{(2x+1)}}{(x+2)\underline{\cancel{(x-1)}}}}_{M1} \quad -(1-x)$$

M1 M1 M1

$$\frac{-3}{(x+2)(x+1)}$$

A1

....

(4 marks)

Write

$$\frac{4x^2-9}{6x+9}\times\frac{2x}{x^2-3x}$$

in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers

$$\frac{\cancel{(2x+3)}(2x-3)}{3\cancel{(2x+3)}}$$

M1

M1

$$\frac{2x-3}{3}\times\frac{2x}{x(x-3)}$$

$$\frac{4x^{\cancel{2}}-6\cancel{x}}{3x^{\cancel{2}}-9\cancel{x}}=\frac{4x-6}{3x-9}$$

A1

....

(3 marks)

Q11. AtoZrevision.com

Show that $\frac{6x^3}{(9x^2-144)} \div \frac{2x^4}{3(x-4)}$ can be written in the form $\frac{1}{x(x+r)}$ where r is an integer.

$$\underbrace{\frac{\cancel{6}^{3}\cancel{x^3}}{(3x+12)\cancel{(3x-12)}}}_{M1} \times \underbrace{\frac{\cancel{3x-12}}{\cancel{2}x^{\cancel{4}}}}_{M1}$$

$$\frac{\cancel{3}}{\cancel{3}x+1\cancel{2}} = \frac{1}{x+4} \quad A1$$

....

(3 marks)

Q12. AtoZrevision.com

Show that $\frac{7x-14}{x^2+4x-12} \div \frac{x-6}{x^3-36x}$ simplifies to ax where a is an integer.

M1 FOR CANCELLING

$$\underbrace{\frac{7\cancel{(x-2)}}{\cancel{(x-2)}\cancel{(x+6)}}}_{M1} \times \underbrace{\frac{x\cancel{(x+6)}\cancel{(x-6)}}{\cancel{x-6}}}_{M1}$$

CANCELS TO $7x$!

A1

....

(4 marks)

Simplify fully

M1

$$\frac{9x^2-1}{3x^2+2x-1} \div \frac{3x+1}{x-2}$$

$$\frac{\cancel{(3x+1)}(3x-1)}{\cancel{(3x+1)}(x-1)} \times \frac{x-2}{3x+1}$$

M1 M1

$$= \frac{(3x-1)(x-2)}{(x-1)(3x+1)}$$

A1

$$= \frac{x-2}{x+1}$$

A1

....

(5 marks)

Show that

$$\frac{1}{2x^2 + x - 15} \div \frac{1}{3x^2 + 9x}$$

simplifies to $\frac{ax}{bx+c}$ where a, b and c are integers to be found.

$$\frac{1}{(2x-5)\cancel{(x+3)}} \times \frac{3x\cancel{(x+3)}}{1}$$

M1 M1

$$\frac{3x}{2x-5}$$

A1

....

(3 marks)

Simplify fully

$$\frac{4x^2+19x-5}{9x^2-16} \div \frac{x+5}{3x-4}$$

M1

$$\frac{(4x-1)\cancel{(x+5)}}{(3x+4)\cancel{(3x-4)}} \times \frac{\cancel{3x-4}}{\cancel{x+5}}$$

M1

M1

M1 FOR AN ATTEMPT AT CANCELLING.

$$\frac{4x-1}{3x+4}$$

A1

....

(5 marks)

Show that

$$\frac{x^4}{x+4} \times \frac{x+2}{x} \div \frac{x^2}{3x+12}$$

simplifies to the form $ax^2 + bx$ where a and b are integers.

M1 for cancelling down

$$\frac{\cancel{x^4}^{3}(x+2)}{\cancel{x}\,\cancel{(x+4)}} \times \frac{3\cancel{(x+4)}}{x^{\cancel{2}}}$$ M1

$$\frac{3\cancel{x^2}\,x\,(x+2)}{\cancel{x}}$$ M1

$$3x(x+2)$$

$$= 3x^2 + 6x$$ A1

....

(4 marks)

Q17.

Simplify fully

$$\frac{8a}{3a+6} \times \frac{5a+10}{3a^2} \div \frac{4}{15a^3}$$

$$\frac{\cancel{8}a}{3\cancel{(a+2)}} \times \frac{5\cancel{(a+2)}}{\cancel{3}\cancel{a^2}} \times \frac{\cancel{15}\cancel{a^3}}{\cancel{4}}$$

M1

A1 $$\frac{2a}{3} \times 5 \times 5a$$

M1

FOR CANCELLING

$$\frac{50a^2}{3}$$

A1

....

(4 marks)

Write

$$5-(x+2)\div\left(\frac{x^2-4}{x-3}\right)$$

as a single fraction. Simplify your answer fully.

$$5-(x+2)\times\frac{x-3}{(x-2)(x+2)} \quad B_1$$

M_1

$$\frac{5(x-2)(x+2)}{(x-2)(x+2)}-(x+2)\times\frac{x-3}{(x-2)(x+2)}$$

$$\frac{5(x^2-4)}{x^2-4}-\frac{(x+2)(x-3)}{x^2-4}$$

M_1

$$\frac{4x-7}{x-2} \quad A_1$$

....

(4 marks)

Show that $3-(x-1)\div\left(\frac{x^2-1}{3x+2}\right)$ can be written as $\frac{a}{x+b}$ where a and b are integers.

$$\cancel{x-1} \times \frac{(3x+2)}{(x+1)\cancel{(x-1)}}$$ M1

$$3-\frac{(3x+2)}{x+1}$$ M1

$$\frac{3(x+1)-3x+2}{(x+1)}$$ M1

$$=\frac{1}{x+1}$$ A1

....

(4 marks)

Show that $6+\left[(x+5)\div\frac{x^2+3x-10}{x-1}\right]$ simplifies to $\frac{ax-b}{cx-d}$ where a, b, c and d are integers.

$$\cancel{(x+5)} \times \frac{x-1}{\cancel{(x+5)}(x-2)}$$ M1

$$6+\frac{x-1}{x-2}$$ M1

$$\frac{6(x-2)+x-1}{x-2}$$

$$=\frac{6x-12+x-1}{x-2}$$

M1 SIMPLIFYING

$$=\frac{7x-13}{x-2}$$ A1

....

(4 marks)

Q21.

Express

$$\left(\frac{4}{2x-5} - \frac{3}{2x-3}\right) \div \frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

$$\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$ M1

M1

$$\frac{\cancel{2x+3}}{(\cancel{2x-5})(2x-3)} \times \frac{(3x-1)(\cancel{2x-5})}{x(3-2x)(\cancel{3+2x})}$$

M1 SIMPLIFYING

$$\frac{3x-1}{x(2x-3)(3-2x)}$$

A1

....

(4 marks)

There are only n red balls and $(n+1)$ blue balls in a bag.

Shamsa takes at random 2 balls from the bag.

Find an expression for the probability that both balls are the same colour.

HAVING DONE ALGEBRAIC TREE DIAGRAMS WOULD HELP WITH THIS.

TWO RED : $\frac{n}{2n+1} \times \frac{n-1}{2n}$ (1)

TWO BLUE : $\frac{n+1}{2n+1} \times \frac{n}{2n}$ (2)

M1

(1) + (2) $= \frac{n(n-1)}{2n(2n+1)} + \frac{n(n+1)}{(2n+1)(2n)}$ A1

M1

$= \frac{n^2 - n + n^2 + n}{}$

$= \frac{\cancel{2}n^{\cancel{2}}}{\cancel{2n}(2n+1)} = \frac{n}{2n+1}$ A1

……………………………………

(4 marks)

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

What is the result when we simplify the expression:

$$\left(1+\frac{1}{x}\right)\left(1-\frac{2}{x+1}\right)\left(1+\frac{2}{x-1}\right) =$$

$$\left(\frac{x+1}{x}\right)\left(\frac{x+1-2}{x+1}\right)\left(\frac{x-1+2}{x-1}\right) =$$

$$\left(\frac{x+1}{x}\right)\left(\frac{x-1}{x+1}\right)\left(\frac{x+1}{x-1}\right) = \frac{x+1}{x}$$

Q2. AtoZrevision.com

When exactly is the value of the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{n}\right)$ equal to an integer?

THIS LOOKS FANCY, BUT IS ACTUALLY:

$$\left(\frac{\cancel{3}}{2}\right)\left(\frac{4}{\cancel{3}}\right)\left(\frac{\cancel{5}}{4}\right)\left(\frac{6}{\cancel{5}}\right)\left(\frac{n+1}{n}\right)$$

NUMERATORS CANCEL WITH FOLLOWING DENOMINATORS - EXCEPT $\frac{n+1}{2}$

$\therefore$ ONLY AN INTEGER WHEN

when n is odd

when n is even

when n is a multiple of 3

always

never