NORMAL DISTRIBUTIONS

AtoZrevision

THE NORMAL DISTRIBUTION. CALCULATING THE MEAN AND STANDARD DEVIATION. HISTOGRAMS AND PERCENTILES.

MARK SCHEME

Q1. AtoZrevision.com

$\frac{\sum x^2}{12} - ('2.8')^2$ (with/without square root, and Σx^2 in range 70~120 incl.)		M1
$= 0.42356...$	awrt 0.4 (but '0.4' only is A0)	A1 (4)
'2.8' ± 2 × '0.4'		M1
(Only '3.71' is outside this range so $\frac{11}{12} \times 100 =$)	92% awrt	A1ft (2)
Close to 95% so normal model <u>is</u> supported	OR not 95% so normal <u>not</u> supported	M1 A1ft (2) [8]

(a)(ii) M1 Allow Σx^2 in given range and ft their mean. (Note correct $\Sigma x^2 = 95.6179$)

Answer is given in the question.

A1 for **complete calculation seen** including square root, leading to **awrt** 0.4

(b) M1 attempts limit(s) using their mean and their 0.4 or better

(correct limits using 1dp values are 2.0 and 3.6)

A1ft allow ft percentages, awrt nearest integer

Note:

Number of outliers	1	2	3	4	… etc
Percentage within 2 s.d.	92	83	75	67	… etc

(ordered values are:
2.09 2.3 2.45 2.61 2.61 2.65 2.86 2.87 2.96 3.1 3.28 3.71)

(c) M1 for comparison with 95%

A1ft for conclusion consistent with comparison. Follow through their (b)

ALT. (a)(ii) Using deviations from mean:

$\frac{\sum (x - '2.8')^2}{12}$ (with/without square root, and $\Sigma(x - '2.8')^2$ in range) M1

Allow $\Sigma(x - '2.8')^2$ in range 1.5~2.5 inclusive and ft their mean.

(Note correct $\Sigma(x - '2.8')^2 = 2.1539$)

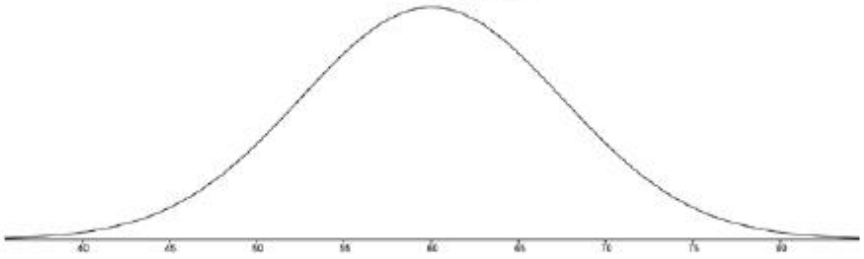
Q2. AtoZrevision.com

B1 B1 B1 B1 for any four from • Histogram will allow you to see if the distribution is bell-shaped • Histogram requires continuous data • Calculation of frequency density is incorrect as class width should be 10.5 – 5.5 (= 5) • 68% of the data should lie within 1 standard deviation of the mean • 95% of the data should lie within 2 standard deviations of the mean	B1 for each assessment (maximum 4) of the appropriateness of using a histogram and calculating means and standard deviations Condone comments which refer to additional calculations that Bien could include to determine whether the data was normally distributed: • Finding the median would also be appropriate • Calculating the skew would be appropriate	**(4)**

Q3. AtoZrevision.com

	B1 Means will be the same/similar B1 Standard deviation of individual values will be greater dB1 So conclusion not supported	B1 for assessment of means B1 for assessment of standard deviations dB1 for conclusion (dep on 2nd B1)	**(3)**

Q4. AtoZrevision.com

Question	Scheme	Marks
(a)		B1 depB1 depB1 (3)
(b)	Time taken. It is continuous data.	B1 B1 (2) [5]
	Notes	
(a)	B1 for a reasonable bell-shaped curve which does not cross x-axis depB1 (dependent on previous B1) for centre on 60 depB1 (dependent on 1st B1) for lower tail ending between 30 and 40 and upper tail ending between 80 and 90	
(b)	Note the two marks are independent	

Q5. AtoZrevision.com

Question	Answer	Additional guidance	Mark
(a)	M1 Test A: $\frac{16.3-14.4}{1.5}$, Test B: $\frac{21.6-19.8}{2.4}$ A1 Test A: 1.26(6…) A1 Test B: 0.75 B1 Better/faster performance in Test B, relative to the other students, with a reason B1 Lower standardised score in Test B, oe	M1 for either correct calculation A1 for 1.26-1.27 A1 for 0.75 B1 for contextual interpretation of results. B0 if no reason. B1 for statistical reasoning, using standardised scores, to support conclusion.	(5)
(b)	M1 for reference to 68% or just over $\frac{2}{3}$ A1 for 34%		(2)

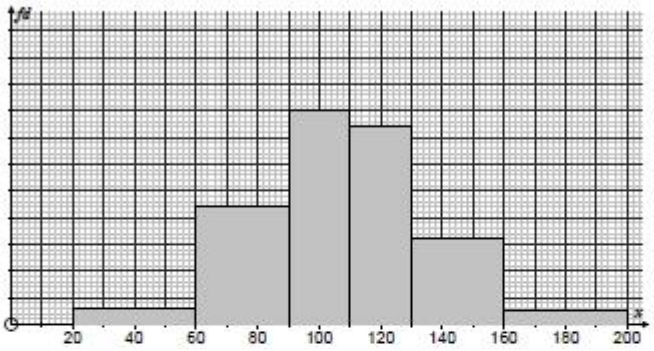
(i)	B1 for e.g. a histogram would allow you to check for a bell shape to the distribution	B1 for correct comment on the appropriateness of using a histogram Accept reference to identifying if the data is skewed or not. Accept for checking if distribution is symmetrical.	(1)
(ii)	B1B1 for each of two from: B1 for e.g. checking that mean, median and mode are equal B1 for e.g. • checking that 68% of data are within 1 standard deviation of the mean • checking that 95% of data are within 2 standard deviations of the mean	B1 for correct comment on the appropriateness of using mean, median and mode Allow mean, median and mode should be similar / close. B1 for correct comment on the appropriateness of using mean with standard deviation.	(2)

Q7. AtoZrevision.com

(a)	B1 Normal (distribution)	B1 for correct distribution	**(1)**
(b)	B2 Temperatures are higher in the summer by 10.5 °C OR Mean temperature in summer is 14 °C which is greater than mean temperature in winter which is 3.5 °C	B2 for a complete comparison with correct figure/figures (accept equivalent or converse statements) (B1 for an incomplete response, e.g. 'higher in summer' with no/incorrect figures or statement of correct figures with no comparison)	**(2)**
(c)	M1 0 to 7 represents , mean ± 3 standard deviations A1 (7÷6 or 3.5 ÷ =3) 1.16(66…) = 1.2 to one decimal place	M1 for recognising that all data lies within 3 standard deviations of mean A1 awrt 1.2 (scores 2 out of 2)	**(2)**
(d)	B1 Winter temperatures have greater spread, or area under each graph is the same (or is 1)	B1 for equivalent comment recognising that greater spread leads to lower height Allow converse statement.	**(1)**
(e)	B2 Sample means will have less spread than daily temperatures, so Carol is not correct.	B2 for a complete answer with correct reasoning. (B1 for an incomplete answer, e.g. correct reasoning with incorrect/no conclusion, or correct conclusion with an attempt at reasoning.)	**(2)**

(f)	B1 15.6 is 2 s.d. above the mean/1.6 represents 2 s.d. B1 0.025 (probability of temperature above 15.6 °C) M1 '0.025' × '0.025' A1ft = 0.000625 (< 0.001) A1ft Greta is correct	B1 for recognising 2 s.d. from mean (may be implied by 2nd B1) B1 for correct probability of tail (allow 0.02275… from calculator) M1 for p^2 with $0 < p < 0.5$ A1ft for awrt 0.0006 (allow awrt 0.0005 from calculator) A1ft (dependent on M1A1) for correct conclusion consistent with their 0.000625	**(5)**

Q8. AtoZrevision.com

Question	Scheme	Marks
(a)	6÷40, etc Frequency densities: (may see multiples of...) 0.15, 1.1, 2.0, 1.85, 0.8, 0.125  	M1 A1 A1 B1 (4)
(b)	$\sqrt{\frac{1794625}{145}-\left(\frac{15535}{145}\right)^2}$ = **29.9**(6978...) or awrt **30.0**	M1 A1 (2)
(c)(i)	2 × s.d. 107 ±2×30 ⇒ **47** and **167** (kg) awrt	M1 M1 A1 (3)
(c)(ii)	(There is evidence that) the Zoologist is <u>correct</u> as most of histogram (or data) in this range / within 2SD of mean **cao**	B1 (1) **[10]**
	Notes	
(a)	**If all bars correct (±½ square tolerance) award M1A1A1 - OVERLAY** Otherwise: **M1** for attempt at least one f ÷ c/w (implied by <u>one</u> correct fd, or by <u>any</u> histogram bar) **A1** if three of their <u>bars</u> correct height **A1** for all bars fully correct **B1** for labels 'frequency density' and 'weight (kg)' (allow fd and *x* as minimum) (Figures on vertical axis are not required.)	
(b)	**M1** for full attempt at s.d. including √ (award M1 if $\sqrt{\text{awrt } 898}$ is seen) **A1** for <u>29.9</u> or better or awrt <u>30.0</u> (NB: **correct working must be seen**) Condone 30 as final answer only **if clearly not rounded** from an incorrect intermediate answer. Condone missing √ sign **if** clear working for variance and answer is <u>29.9</u> or better	
(c)(i)	**1st M1** for 2 × s.d. **2nd M1** for 2 × 30 applied to mean. M1M1 can be implied by awrt (47 or 167) seen **A1** for awrt 47 and awrt 167 (either order)	
(ii)	Require correct conclusion <u>AND</u> a sensible reason <u>from graph/data</u>: Accept: (about) 95% in range / within 2SD of mean / within these values Condone: nearly all (or 99.8%) within 3SD of mean / between 17 and 197 Condone: bell-shaped / symmetrical for B1, **BUT:** most values in middle **is B0**	

Q9. AtoZrevision.com

	5ST1H_01 Scheme	Marks
(a)	Mean = 55 (85-'55')/3 or (85-25)/6 = 10	B1 M1 A1
(b)	(Test 1) $\frac{60-'55'}{'10'}=0.5$ (Test 2) $\frac{60-64}{12}=-0.3333...$ Performed better on Test 1 ... as standardised score is higher	(3) M1A1ftA1 B1 dB1 (5) **[8]**
	Notes	
(a) **(b)**	B1 allow anything [53,57] M1 for finding half the range [27, 33] and using 3sd or finding the range [54, 66] and using 6sd M1 $\frac{60-'55'}{'10'}$ or $\frac{60-64}{12}$ A1 for [9,11] A1ft for Test 1 correct to 1dp or better using their values from (a) A1 for -0.3 or better 1st B1 performed better on Test 1 2nd B1 dependent on first B1 for Test 1 score is higher OR Test 1 score is positive **and** Test 2 score is negative OR Test 1 is above mean **and** Test 2 is below mean (condone average)	

Q10. AtoZrevision.com

Question number	Answer	Additional guidance	Mark
(a)	B1 3.0	B1 allow 3	(1)
(b)	B1 55%	B1 allow 0.55	(1)
(c)	B1 distribution is symmetric		(1)
(d)	B1 distribution symmetric so $\frac{5.7+5.3}{2}=5.5$ B1 (95% of data should fall between) $5.5 \pm 2\times0.75$ (from 4 to 7) B1 97.5% – 2.5% = middle 95% dB1 so claim is supported	B1 for using symmetry to identify the mean Allow this B mark for demonstration that two appropriately chosen percentiles are equidistant from 5.5 e.g. $6.1-5.5=0.6$ and $5.5-4.9=0.6$ B1 for use of mean +/- 2 s.d. Must show calculation using 2×0.75 not just 4 and 7 B1 for comparing boundaries with data dB1 for identifying claim is supported (dep on 1st & 2nd B1)	(4)

Question number	Answer	Additional guidance	Mark
(a)	M1 for standardising $\frac{37-25}{6}$ $(=2)$ oe **OR** by inspection stating 37 minutes is 2 sd's above 25 minutes. [May be seen on a diagram] M1 for standardising $\frac{19-25}{6}$ $(=-1)$ oe **OR** by inspection stating 19 minutes is 1 sd below 25 minutes. [May be seen on a diagram] M1 for either 0.95 ÷ 2 (=0.475) or 0.68 ÷ 2 (= 0.34) A1 0.815 or 81.5% depB1ft 0.815 > 0.8 so Shanaya is correct	M1 for working out 37 is 2 s.d. above the mean. Allow M1 for $25+\mathbf{2}\times 6 = 37$ M1 for working out 19 is 1 s.d. below the mean. A Allow M1 for $25-(\mathbf{1})\times 6 = 19$ M1 for correct use of either awrt 0.67 or 0.68 or awrt 67% or 68% **OR** for the correct use of either 0.95 or 95% i.e. 95% ÷ 2 **OR** 67/8% ÷ 2 A1 – for use of both 47.5% or 0.475 **AND** 34% or 0.34 AND adding them to achieve 81.5% or 0.815 Allow answers greater than 81% or less than and equal to 82%. Note: Value from calculator is 81.8% Sight of 0.815 or 81.5% with no working is M1M1M1A1 Dependant on getting **any one** of the M marks. depB1ft for correct conclusion based on **their** evaluated probability between 0 and 100%. Follow through their value. Conclusion required for this mark.	(5)
(b)	B1 Data [positively] skewed/not symmetrical Accept graph not symmetrical	B1 for a correct comment explaining why a normal distribution is not an appropriate model in a skewed distribution. Do not accept Negative Skew.	(1)
(c)	M1 bell shaped curve centred on 25 within half of a square A1 Tails ending at 7 and 43 within half of a square	There must be an attempt at a symmetrical bell-shaped curve centred around 25.	(2)
(d)	M1 0.6 + 0.3 – 0.15 A1 0.75	M1 for correct use of P(A or B) = P(A) + P(B) – P(A and B)	(2)

Q12. AtoZrevision.com

Question	Scheme	Marks
(i)	Normal (distribution) or C	B1
	Time taken is a **continuous** variable	B1
		(2)
(ii)	Discrete uniform (distribution) or A	B1
	Each digit is equally likely	B1
		(2)
		[4]
	Notes	
(i)	2nd B1: Must have reference to continuous	
(ii)	1st B1: Condone 'discrete' or 'uniform' on their own 2nd B1: Must have reference to equally likely, o.e.	

Q13. AtoZrevision.com

The gymnast did better on the balance beam, since the standardised score is higher.	B1ft B1ft (2)
$\frac{15.3-14.5}{0.6} = (1.333...)$	M1
Normal distribution has 95% of data within ±2 standard deviations. Since no data is more than 1.3 standard deviations above the mean, it would <u>not</u> be <u>suitable</u> to use a normal distribution to model these data.	M1 A1 (3) [7]

Notes

M1 for using $\frac{\pm(X-\mu)}{\sigma}$

A1 for awrt –0.7 allow –2/3 or $-0.\dot{6}$ for A1

1st B1 for better on the balance beam
2nd B1 for standardised score on balance beam is higher
or standardised score is positive for the balance beam and negative for the vault
or scored above mean on balance beam and below mean on vault
If their (a) > 0.5, then ft vault for both B1 marks.

1st M1 for calculating the standardised score for 15.3 or calculating $14.5 + 2 \times 0.6$ (=15.7)
2nd M1 for use of 95% within ±2 standard deviations of mean/virtually all data within ±3 standard deviations of mean
3rd A1 dependent upon both M marks for correct conclusion, it is <u>not suitable,</u> with correct figures.

e.g. M1 LB = mean – 6.29 (= 48.62) M1 UB = mean + 6.29 (= 61.2) M1 (mean – 52) × 2.6 + (52 – LB) × 2.2 (= 15.002) (57 – mean) × 2.6 + (61.2 – 57) × 2 (= 13.834) '15.002' + '13.834' (= 28.836) M1 28.836 ÷ 43 A1 = 0.671	M1 for correct method to find number of presidents 1 sd below mean M1 for correct method to find number of presidents 1 sd above mean M1 for correct method to find total number of presidents within 1 sd of mean M1 for correct method to find proportion of presidents within 1 sd of mean A1 for answers rounding to 0.67 or 67%	(5)
B1 for e.g. yes, close to 68%	B1 for referring to 68%	(1)