PARALLEL AND COLLINEAR VECTORS

GRADE 8

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions may go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

Q1.

D, E and F are three points on a straight line such that:

$\overrightarrow{DE} = 3e + 6f$

$\overrightarrow{EF} = -10.5e - 21f$ $\quad 3.5(3e+6f)$ M1

Find the ratio of length of DF : length of DE

$1 : 3.5$ A1

....

(2 marks)

Q2.

A, B and C are three points such that:

$\overrightarrow{AB} = 3a + 4b$

$\overrightarrow{AC} = 15a + 20b$ $\quad 5(3a+4b)$ M1 $\therefore$

Prove that A, B and C lie on a straight line.

VECTORS ARE PARALLEL.

THEY ALSO SHARE A COMMON POINT

$\therefore$ THEY LIE ON A STRAIGHT LINE.

....

C1 CONCLUSION

(2 marks)

Q3. AtoZrevision.com

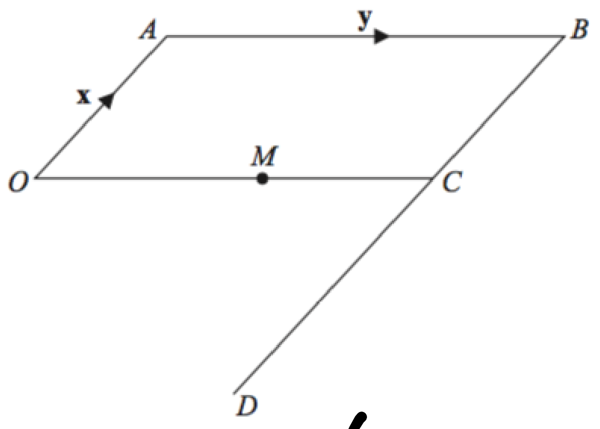


Diagram **NOT** accurately drawn

$\overrightarrow{AM} = \frac{1}{2}y - x$ and $\overrightarrow{OD} = y - 2x \rightarrow 2\left(\frac{1}{2}y - x\right)$

Are these two vectors parallel?

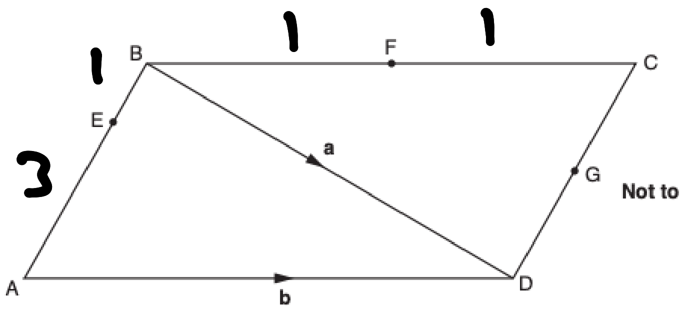
YES

....

(1 mark)

Q4. AtoZrevision.com

$ABCD$ is a parallelogram.



Not to scale

$\overrightarrow{BD} = a$ and $\overrightarrow{AD} = b$.

F is the midpoint of BC. G is the midpoint of DC.
$AE = 3EB$.

Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel. M1

$\therefore$ PARALLEL A1

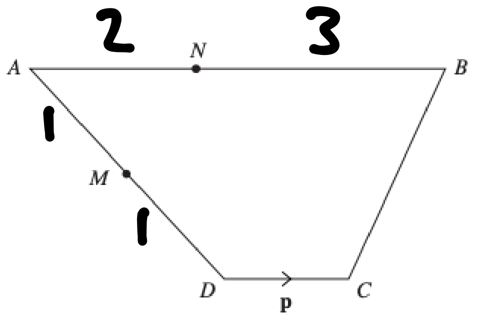
$\overrightarrow{AG} = \frac{3}{2}b - \frac{1}{2}a$ M1 $\frac{1}{2}\left(\frac{3}{2}b - \frac{1}{2}a\right)$

$\overrightarrow{EF} = \frac{1}{4}(-b - a) + b$

$= -\frac{1}{4}b - \frac{1}{4}a + b = \frac{3}{4}b - \frac{1}{4}a$

([illegible] marks)

AB is parallel to DC.



Not drawn accurately

$\overrightarrow{AB} = 5p$
$\overrightarrow{DC} = p$
$\overrightarrow{DA} = 2q - p$

M is the midpoint of AD.
$\overrightarrow{AN} : \overrightarrow{NB} = 2:3$

Show that MN is parallel to CB.

$\overrightarrow{MN} = \frac{1}{2}(2q - p) + 2p$

M1 $= q \cancel{- \frac{1}{2}p + 2p} + 1.5p$

$\overrightarrow{CB} = -p + 2q - p + 5p$

$= 3p + 2q$

M1 $= 2(\underline{1.5p + q})$

$\therefore$ PARALLEL A1

(3 marks)

Q6.

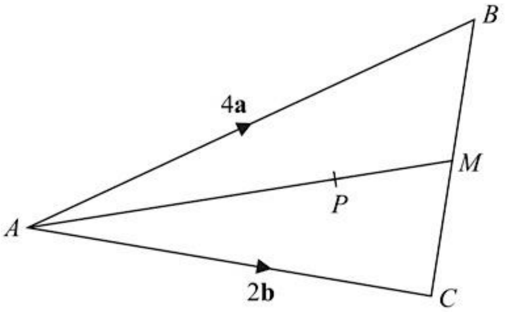


Diagram **NOT** accurately drawn

ABC is a triangle.
The midpoint of BC is M.
P is a point on AM.

$\overrightarrow{AB} = 4a$

$\overrightarrow{AC} = 2b$

$\overrightarrow{AP} = \frac{3}{2}a + \frac{3}{4}b$

Show that AP is parallel to PM

$$\overrightarrow{AP} = \tfrac{3}{2}a + \tfrac{3}{4}b = 3\left(\underline{\tfrac{1}{2}a + \tfrac{1}{4}b}\right)$$

$$\overrightarrow{PM} = -\tfrac{3}{2}a - \tfrac{3}{4}b + 4a + \tfrac{1}{2}(2b - 4a) = \underline{\tfrac{1}{2}a + \tfrac{1}{4}b}$$

M1 A1

$\therefore$ PARALLEL. A1

(3 marks)

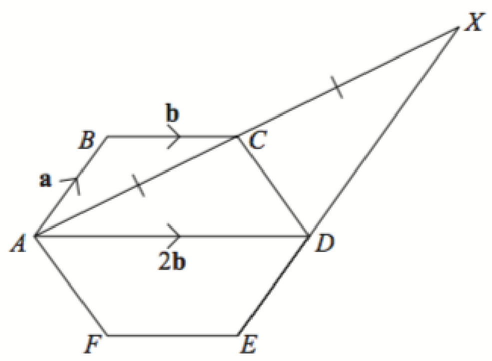


Diagram **NOT** accurately drawn

ABCDEF is a regular hexagon.

$\overrightarrow{AB} = a$

$\overrightarrow{BC} = b$

$\overrightarrow{AD} = 2b$

$\overrightarrow{AC} = \overrightarrow{CX}$

Prove that AB is parallel to DX.

$\overrightarrow{AB} = \underline{a}$

$\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$

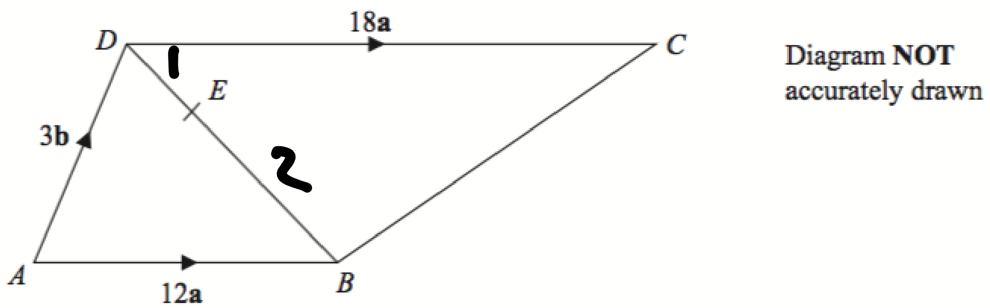
$= -2b + 2(a+b)$ M1

$= 2a$ A1

$= 2(\underline{a})$

$\therefore$ PARALLEL A1

(3 marks)



ABCD is a trapezium.
AB is parallel to *DC*.

$\overrightarrow{AB} = 12a$

$\overrightarrow{AD} = 3b$

$\overrightarrow{DC} = 18a$

E is the point on the diagonal DB such that $DE = \frac{1}{3}DB$.

Show by a vector method that BC is parallel to AE.

$\overrightarrow{BC} = -12a + 3b + 18a$

$= 6a + 3b$ M1

$\overrightarrow{AE} = 3b + \frac{1}{3}(-3b + 12a)$

$= \underline{2b + 4a}$ M1

$\overrightarrow{BC} = \frac{3}{2}(\underline{2b + 4a})$

$\therefore$ PARALLEL A1

(3 marks)

Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix} + b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

$$a\begin{pmatrix}2\\6\end{pmatrix} + b\begin{pmatrix}8\\2\end{pmatrix} = k\begin{pmatrix}13\\6\end{pmatrix}$$

EITHER

$\therefore\ 2a + 8b = 13k$

$6a + 2b = 6k$

M1

$\times 3$

$6a + 24b = 39k$

$-\ 6a + 2b = 6k$

$22b = 33k$

P1 PROCESS TO SOLVE

$b = 1.5k$

$6a + 3k = 6k$

$6a = 3k$

$a = \frac{1}{2}k$

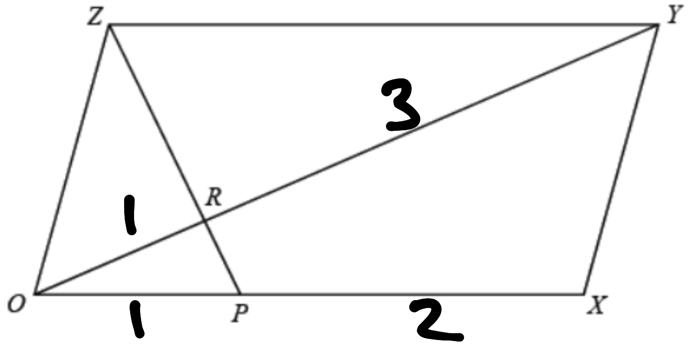
$\therefore\ b = 3 \times a$

$b = 3a$

A1

(3 marks)

$OXYZ$ is a parallelogram.



$\overrightarrow{OX} = \mathbf{a}$
$\overrightarrow{OY} = \mathbf{b}$
P is the point on OX such that $OP : PX = 1 : 2$
R is the point on OY such that $OR : RY = 1 : 3$
Work out, in its simplest form, the ratio $ZP : ZR$

$\overrightarrow{OP} = \frac{1}{3}a \quad \overrightarrow{OR} = \frac{1}{4}b$ P1

$\overrightarrow{ZO} = \overrightarrow{YX} = a - b$ P1

$\overrightarrow{ZP} = a - b + \frac{1}{3}a = \frac{4}{3}a - b$

$\overrightarrow{ZR} = a - b + \frac{1}{4}b = a - \frac{3}{4}b$

EITHER P1

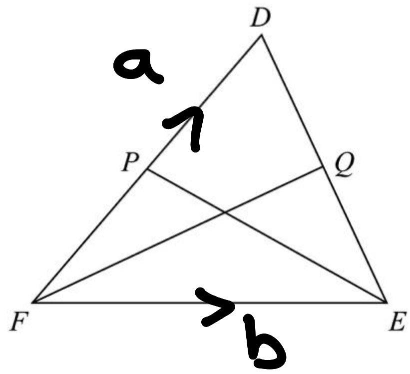
$\overrightarrow{ZP} = \frac{4}{3}\left(a - \frac{3}{4}b\right)$ P1

$\therefore \underline{4 : 3}$ A1

(5 marks)

Q11.

DEF is a triangle.



P is the midpoint of FD.
Q is the midpoint of DE.

$\overrightarrow{FD} = a$ and $\overrightarrow{FE} = b$

Use a vector method to prove that PQ is parallel to FE.

$\overrightarrow{DQ} = \frac{1}{2}(b-a)$

$\overrightarrow{EQ} = \frac{1}{2}(a-b)$

M1 EITHER

$\overrightarrow{PQ} = \frac{1}{2}a + \overrightarrow{DQ} = \frac{1}{2}a + \frac{1}{2}(b-a)$ M1

$\therefore \overrightarrow{PQ} = \frac{1}{2}b$ B1

$\overrightarrow{FE} = b$

$\therefore \overrightarrow{FE} = 2(\overrightarrow{PQ})$ C1

$\therefore$ THEY ARE PARALLEL.

(4 marks)

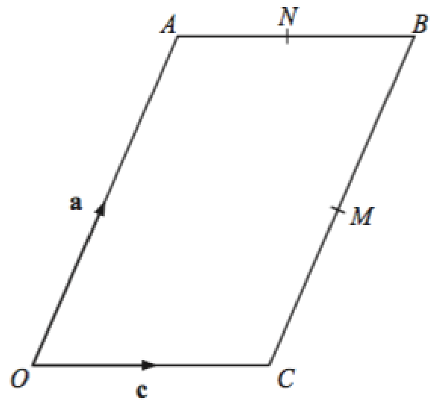


Diagram **NOT** accurately drawn

OABC is a parallelogram.

M is the midpoint of CB.
N is the midpoint of AB.

$\overrightarrow{OA} = a$

$\overrightarrow{OC} = c$

Show that CA is parallel to MN.

$\overrightarrow{CA} = a - c$ B1

$\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{CB}) + \frac{1}{2}(\overrightarrow{BA})$

$= \frac{1}{2}a - \frac{1}{2}c$ B1

$\therefore \overrightarrow{CA} = 2\left(\frac{1}{2}a - \frac{1}{2}c\right)$

$\therefore$ THEY ARE PARALLEL. C1

(3 marks)

Q13.

The diagram shows trapezium *ABCD*.

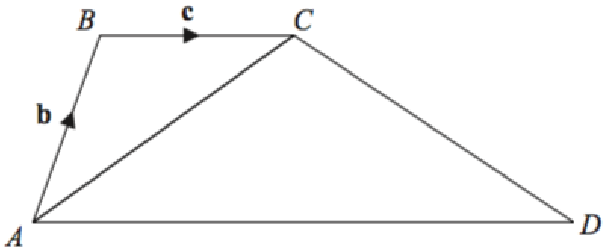


Diagram **NOT** accurately drawn

BC is parallel to *AD*
AD = 3BC

$\overrightarrow{AB} = b, \quad \overrightarrow{BC} = c$

It can be shown that $\overrightarrow{CD} = 2c - b$

The point *P* lies on the line *AC* such that *AP* : *PC* = 2 : 1

Is *BPD* a straight line?

$\overrightarrow{BP} = \overrightarrow{BA} + \frac{2}{3}\overrightarrow{AC}$ (1)

① EITHER M1

(2) $= -b + \frac{2}{3}(b+c)$

$= \frac{2}{3}c - \frac{1}{3}b$

② EITHER M1

$\overrightarrow{PD} = \frac{1}{3}\overrightarrow{AC} + \overrightarrow{CD}$ (1)

(2) $= \frac{1}{3}(b+c) + 2c - b$

$= \frac{7}{3}c - \frac{2}{3}b$

$\overrightarrow{PD}$ IS NOT A MULTIPLE OF $\overrightarrow{BP}$ ∴ THEY ARE NOT PARALLEL. BPD IS NOT A STRAIGHT LINE.

A1

M1

(4 marks)

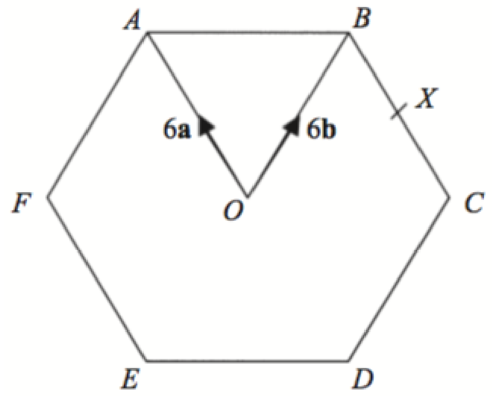


Diagram **NOT** accurately drawn

The diagram shows a regular hexagon $ABCDEF$ with centre O.

$\overrightarrow{OA} = 6a \qquad \overrightarrow{OB} = 6b$

X is the midpoint of BC.

Y is the point on AB extended, such that $AB: BY = 3: 2$

Prove that E, X and Y lie on the same straight line.

$\overrightarrow{AY} = \frac{5}{3}\overrightarrow{AB} \qquad \overrightarrow{BY} = \frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{EY} = 16b - 4a \qquad \overrightarrow{XY} = 4b - a$

ANY

B1

$\overrightarrow{EY} = \frac{4}{3}\overrightarrow{XY}$

$\overrightarrow{EX} = \frac{3}{3}\overrightarrow{XY}$

B1

$\therefore$ PARALLEL

E = COMMON POINT

$\therefore E, X, Y$ LIE ON THE SAME STRAIGHT LINE

B1

(3 marks)

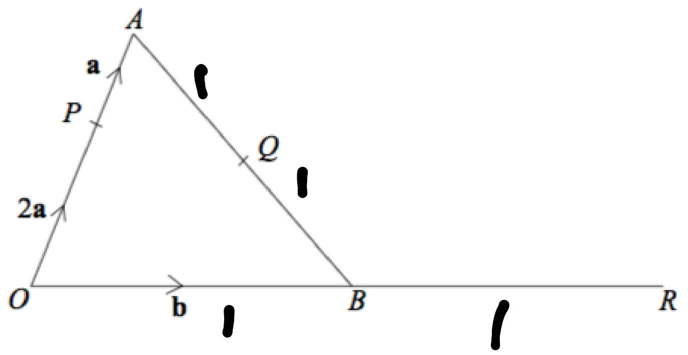


Diagram **NOT** accurately drawn

OAB is a triangle.
B is the midpoint of OR.
Q is the midpoint of AB.

$\overrightarrow{OP} = 2a$

$\overrightarrow{PA} = a$

$\overrightarrow{OB} = b$

Explain why PQR is a straight line.

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$= a + \frac{1}{2}(-3a + b)$

$= -\frac{1}{2}a + \frac{1}{2}b$ M1

$\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR}$ M1

$= \frac{1}{2}(-3a + b) + b$

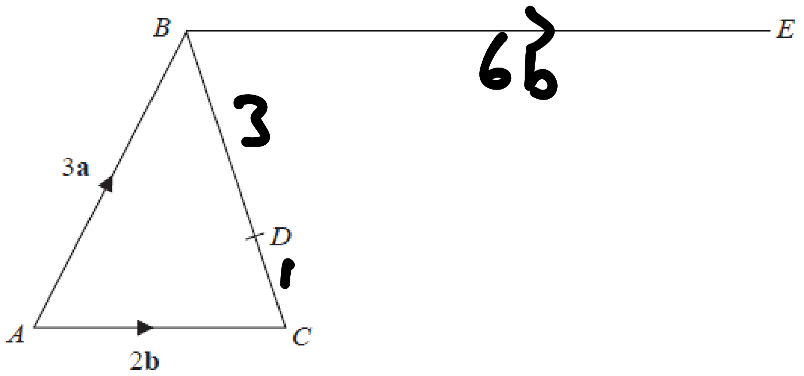
$= -\frac{3}{2}a + \frac{3}{2}b$

$= 3\left(-\frac{1}{2}a + \frac{1}{2}b\right)$ C1

$\therefore$ THEY ARE PARALLEL AND SHARE A COMMON POINT.

(3 marks)

Q16.



The diagram shows triangle *ABC*.

$\overrightarrow{AB} = 3a$

$\overrightarrow{AC} = 2b$

$\overrightarrow{BE} = 3\,\overrightarrow{AC}$

D is the point on *BC* such that *BD* : *DC* = 3 : 1

Prove that *ADE* is a straight line.

$$\overrightarrow{AC} + \tfrac{1}{4}\left(\overrightarrow{CB}\right)$$

$$\overrightarrow{AD} = 2b + \tfrac{1}{4}(-2b + 3a) \quad M1$$

$$= \tfrac{3}{4}(\underline{2b + a})$$

$$\overrightarrow{DE} = \tfrac{3}{4}\left(\overrightarrow{CB}\right) + \overrightarrow{BE} \quad M1$$

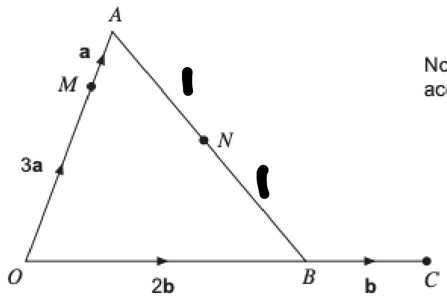
$$= \tfrac{3}{4}(-2b + 3a) + 6b$$

$$= \tfrac{9}{4}(\underline{2b + a}) \leftarrow \quad M1$$

∴ THEY ARE PARALLEL AND SHARE A COMMON POINT. C1

(4 marks)

OAB is a triangle.
OBC is a straight line.



$\overrightarrow{OA} = 4a$
$\overrightarrow{OB} = 2b$
$\overrightarrow{BC} = b$
$\overrightarrow{OM} = 3a$

N is the midpoint of AB.

Show that M, N and C lie on a straight line.

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$$
$$= a + \tfrac{1}{2}(-4a + 2b)$$
$$= -a + b$$

M1

$$\overrightarrow{NC} = \overrightarrow{NB} + \overrightarrow{BC}$$
$$= \tfrac{1}{2}(-4a + 2b) + b$$
$$= -2a + 2b$$
$$= 2(-a + b)$$

M1

M1

VECTORS ARE PARALLEL AND HAVE A COMMON POINT.

C1

(4 marks)

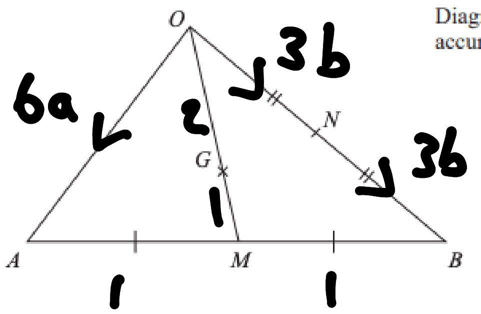


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 6a$ and $\overrightarrow{OB} = 6b$
M is the midpoint of AB.

N is the midpoint of OB.
G is the point on OM such that $OG : GM = 2 : 1$.

Show that AGN is a straight line.

$$\overrightarrow{AG} = -6a + \tfrac{2}{3}\left(6a + \tfrac{1}{2}(-6a+6b)\right)$$ M1

$$= -6a + \tfrac{2}{3}(3a+3b)$$

$$= -6a + 2a + 2b$$

$$* = -4a + 2b = 2(\underline{-2a+b})$$

$$\overrightarrow{GN} = -\tfrac{2}{3}(3a+3b) + 3b$$ M1

$$= -2a - 2b + 3b$$

$$* = \underline{-2a + b}$$

* EITHER M1

$\therefore$ PARALLEL. THEY SHARE A COMMON POINT TOO. C1

(4 marks)

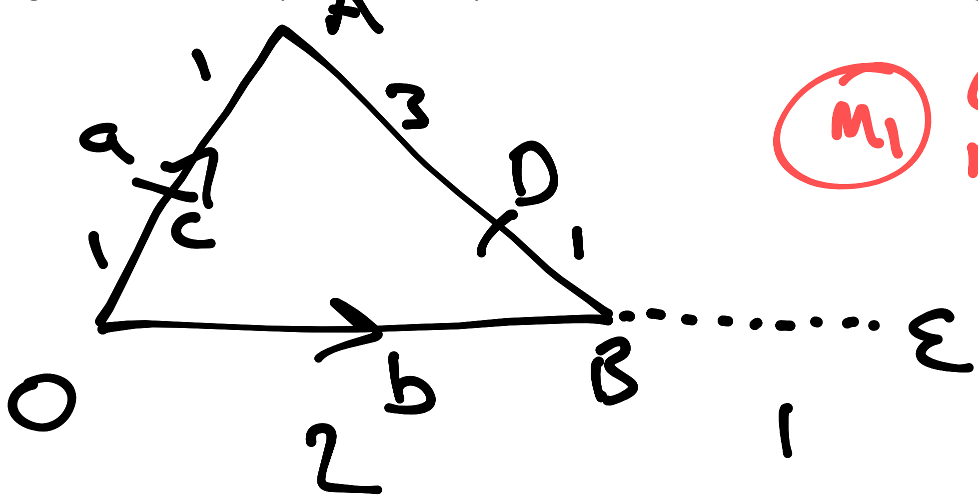
Q19. AtoZrevision.com

OAB is a triangle.

$\overrightarrow{OA} = a \qquad \overrightarrow{OB} = b$

C is the midpoint of OA.
D is the point on AB such that $AD : DB = 3 : 1$
E is the point such that $\overrightarrow{OB} = 2\overrightarrow{BE}$

Using a vector method, prove that the points C, D and E lie on the same straight line.



M1 CORRECT DIAGRAM

$\overrightarrow{CD} = \frac{1}{2}a + \frac{3}{4}(-a+b)$ $= \overrightarrow{OA} + \overrightarrow{AD}$ M1

$* = -\frac{1}{4}a + \frac{3}{4}b$

$\overrightarrow{DE} = \frac{1}{4}(-a+b) + \frac{1}{2}b$ M1

$= \overrightarrow{DB} + \overrightarrow{BE}$

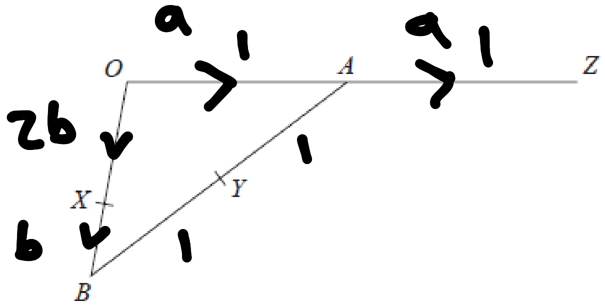
* EITHER

$* = -\frac{1}{4}a + \frac{3}{4}b$ M1

VECTORS $\overrightarrow{CD}$ AND $\overrightarrow{DE}$ ARE PARALLEL AND SHARE A COMMON POINT.

A1

(5 marks)



OAB is a triangle.
A is the midpoint of OZ
Y is the midpoint of AB
X is a point on OB

$\overrightarrow{OA} = a \quad \overrightarrow{OX} = 2b \quad \overrightarrow{XB} = b$

Prove that XYZ is a straight line.

$$\overrightarrow{XY} = \overrightarrow{XB} + \overrightarrow{BY}$$
$$= b + \tfrac{1}{2}(-3b + a)$$
M1
$$= -\tfrac{1}{2}b + \tfrac{1}{2}a$$
A1

$$\overrightarrow{YZ} = \overrightarrow{YA} + \overrightarrow{AZ}$$
$$= \tfrac{1}{2}(-3b + a) + a$$
M1
$$= -\tfrac{3}{2}b + \tfrac{3}{2}a$$
$$= 3\left(-\tfrac{1}{2}b + \tfrac{1}{2}a\right)$$

$\overrightarrow{XY}$ AND $\overrightarrow{YZ}$ ARE PARALLEL AND THEY SHARE A COMMON POINT.
C1

(4 marks)

$ACEF$ is a parallelogram.
B is the midpoint of AC.
M is the midpoint of BE.

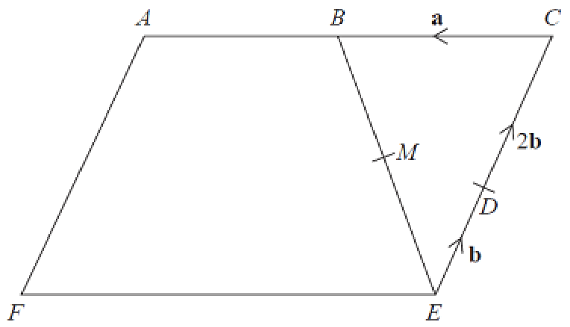


Diagram **NOT** accurately drawn

Prove that AMD is straight line.

$$\overrightarrow{DM} = -b + \tfrac{1}{2}(3b + a) \quad M1$$

$$\overrightarrow{MA} = \tfrac{1}{2}(3b + a) + a \quad M1$$

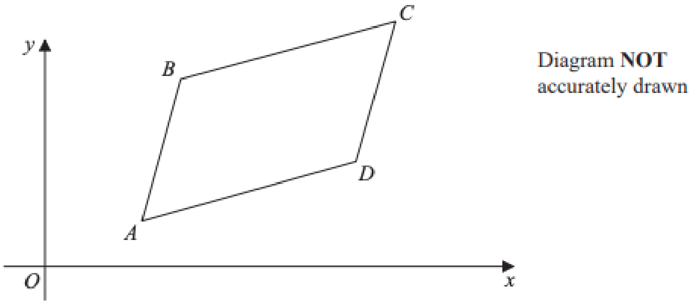
$$\overrightarrow{DM} = \tfrac{1}{2}(a + b) \quad A1$$

$$\overrightarrow{MA} = \tfrac{3}{2}(a + b) \quad A1$$

$\therefore$ THEY ARE PARALLEL AND THEY SHARE A COMMON POINT. C1

(5 marks)

The diagram shows parallelogram $ABCD$.



$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$

The point B has coordinates $(5,8)$

The point E has coordinates $(63,211)$

Use a vector method to prove that ABE is a straight line.

$\overrightarrow{BE} =$

$\begin{pmatrix} 63 \\ 211 \end{pmatrix} - \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{matrix} 58 \\ 203 \end{matrix}$ M1

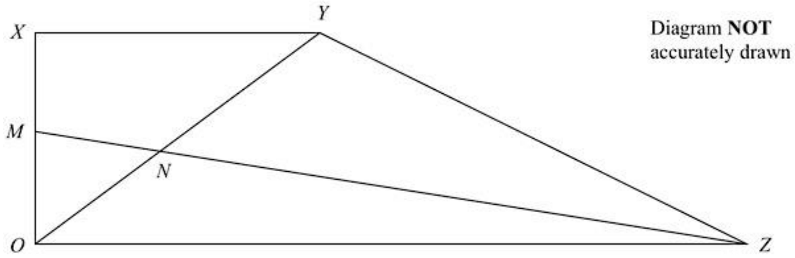
$\frac{58}{2} = 29$ $\frac{203}{7} = 29$

$\overrightarrow{BE} = \underline{29}\begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$\therefore$ THEY ARE PARALLEL AND SHARE A COMMON POINT. A1

(2 marks)

$OXYZ$ is a trapezium.



$\overrightarrow{OX} = a$

$\overrightarrow{XY} = b$

$\overrightarrow{OZ} = 3b$

$\overrightarrow{MZ} = -\frac{1}{2}a + 3b$

M is the midpoint of OX
N is the point such that MNZ and ONY are straight lines.

M1

Given that $ON : OY = \lambda : 1$

Use a vector method to find the value of λ

$\overrightarrow{ON} = \lambda \overrightarrow{OY} = \lambda(a+b) = \lambda a + \lambda b$

M1

ROUTE 2: $\overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN}$

$= \frac{1}{2}a + \mu(-\frac{1}{2}a + 3b)$

$= \frac{1}{2}a - \frac{1}{2}\mu a + 3\mu b$

A1

M1

$\lambda = \frac{1}{2} - \frac{1}{2}\mu \qquad \lambda = 3\mu$

$3\mu = \frac{1}{2} - \frac{1}{2}\mu \quad \therefore \mu = \frac{1}{7}$

$\lambda = 3\mu = \frac{3}{7}$

A1

(5 marks)

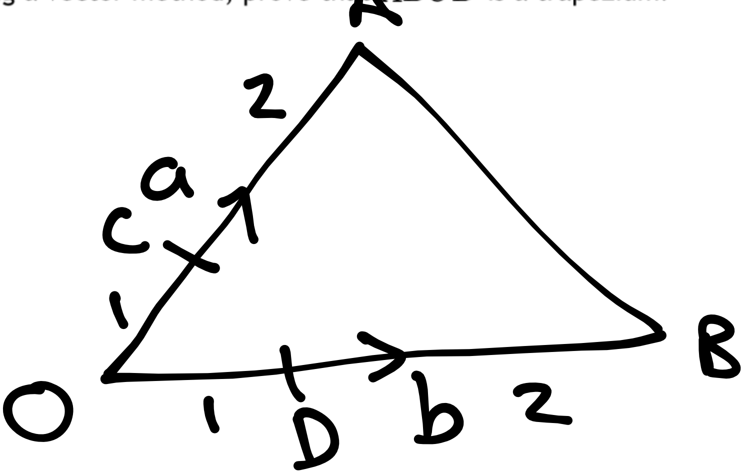
Q24.

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

The point C lies on OA such that $OC : CA = 1 : 2$
The point D lies on OB such that $OD : DB = 1 : 2$

Using a vector method, prove that $ABCD$ is a trapezium.



$\overrightarrow{AB} = -a + b$ M1

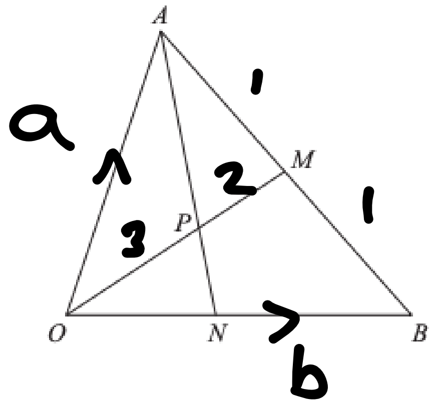
$\overrightarrow{CD} = \frac{1}{3}(-a) + \frac{1}{3}(b)$

$= -\frac{1}{3}a + \frac{1}{3}b$

$= \frac{1}{3}(-a + b)$ M1

ABCD HAS 4 SIDES.
TWO ARE PARALLEL. A1

(3 marks)



OAB is a triangle.
OPM and APN are straight lines.
M is the midpoint of AB.

$\overrightarrow{OA} = a \qquad \overrightarrow{OB} = b$

$OP : PM = 3 : 2$

Prove that ONB is a straight line.

$\overrightarrow{AB} = -a + b$

$\overrightarrow{AM} = \frac{1}{2}(-a + b)$ A1

$\overrightarrow{OM} = a + \frac{1}{2}(-a + b)$

P1 $= \frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{OP} = \frac{3}{5}\left(\frac{1}{2}a + \frac{1}{2}b\right)$

P1 $= \frac{3}{10}a + \frac{3}{10}b$

$\overrightarrow{AP}$ P1 $= -a + \frac{3}{10}a + \frac{3}{10}b$

$= -\frac{7}{10}a + \frac{3}{10}b$

$\overrightarrow{AN} = \lambda\left(-\frac{7}{10}a + \frac{3}{10}b\right) = -a + \mu b$

$-\frac{7}{10}\lambda = -1$ P1

$\lambda = 10/7$

$\frac{3}{10}\lambda = \mu$

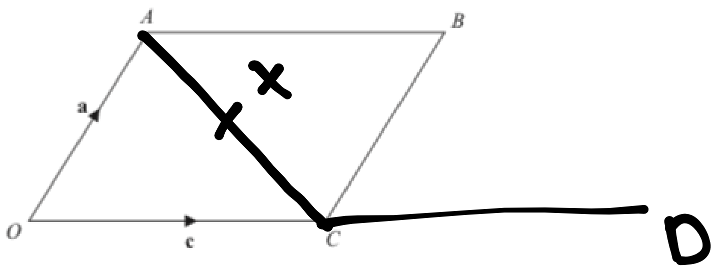
$\mu = 3/7$ *

* $\overrightarrow{ON} : \overrightarrow{NB}$

$3 : 4$

$\therefore$ PARALLEL AND THEY SHARE A COMMON POINT.

(5 marks)



$OABC$ is a parallelogram.

$\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$

X is the midpoint of the line AC.
OCD is a straight line so that $OC: CD = k: 1$

Given that $\overrightarrow{XD} = 3c - \frac{1}{2}a$

Find the value of k.

$\overrightarrow{XD} = 3c - \frac{1}{2}a$

$\overrightarrow{AC} = c - a$ P1

$\overrightarrow{XC} = \frac{1}{2}c - \frac{1}{2}a$

$\overrightarrow{CD} = \overrightarrow{CX} + \overrightarrow{XD}$ P1

$= \frac{1}{2}a - \frac{1}{2}c + 3c - \frac{1}{2}a$

$\overrightarrow{CD} = \frac{5}{2}c$ P1

$\overrightarrow{OC} = c$

RATIO: $1 : \frac{5}{2}$

$2 : 5$ A1

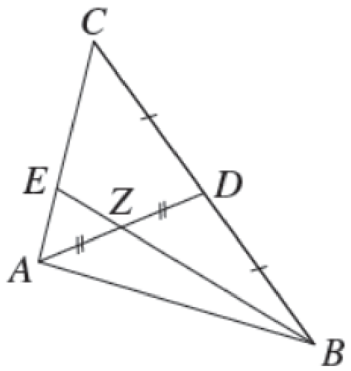
$\frac{2}{5} : 1 \quad \therefore k = \frac{2}{5}$

(4 marks)

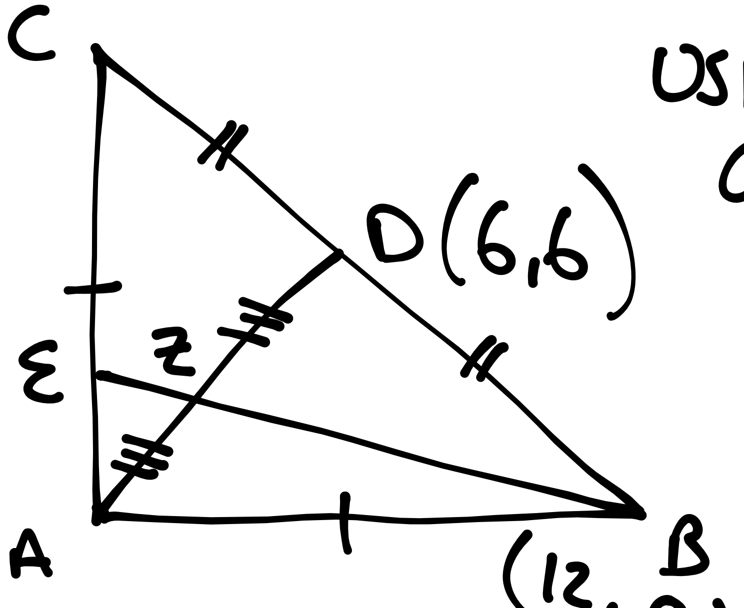
QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

The diagram shows a triangle ABC and two lines AD and BE, where D is the midpoint of BC and E lies on CA. The lines AD and BE meet at Z, the midpoint of AD.



What is the ratio of the length CE to the length EA?



USING COORDINATE GEOMETRY:

AD IS THE LINE $y = x$

$D = (6,6)$ SO $Z = (3,3)$.

$ZB = y = mx + c$

$= y = -3/9\, x + c$ SUB IN $(12,0)$

$c = 4$

$y = -3/9\, x + 4$

$E = (0,4)$

$CE = 8$

$EA = 4$

$8 : 4$

$2 : 1$

PHEW!