# QUADRATIC INEQUALITIES

GRADE 9

AtoZrevision

## Examination Practice Questions

**You should have:**

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

### Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

### Information

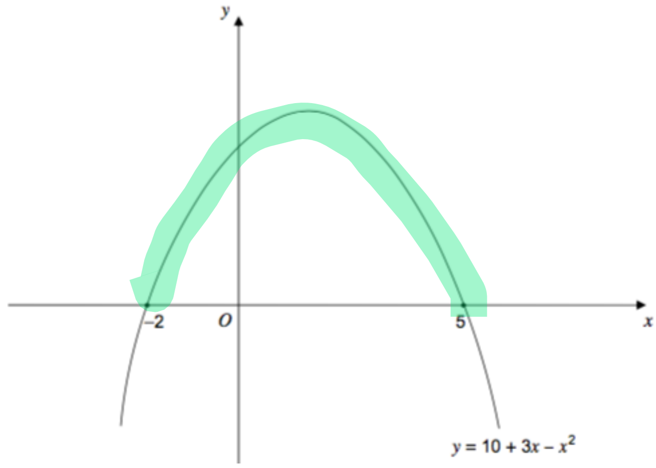
- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

### Advice

- Read each question carefully before you start to answer it.
- Don't forget to have fun.
- Check your answers at the end.

## Q1. AtoZrevision.com

Here is a sketch of $y = 10 + 3x - x^2$




Write down the solution of $10 + 3x - x^2 \geq 0$

M1 2 AND 5

A1

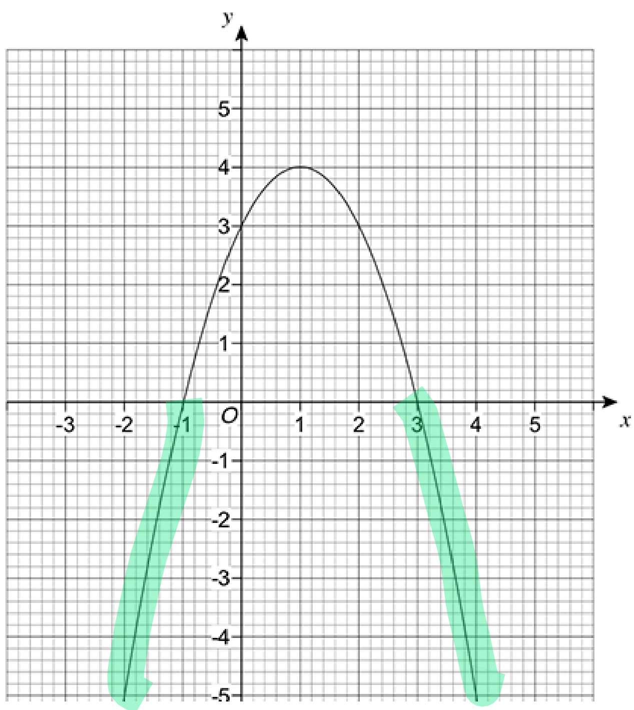
$2 \leqslant x \leqslant 5$

**(2 marks)**

## Q2. AtoZrevision.com

$f(x)$ is a quadratic function with domain all real values of $x$.

Part of the graph of $y = f(x)$ is shown.




Use the graph to solve $f(x) < 0$

A1

$-1 > x \quad x > 3$

A1

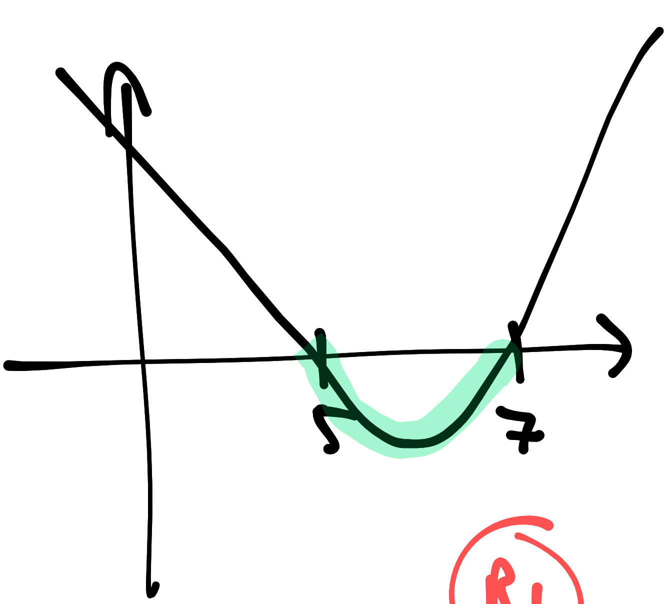
**(2 marks)**

**Q3.** AtoZrevision.com

Solve the inequality $x^2 - 12x + 35 \leq 0$.

M1 $(x-5)(x-7)$

$x = 5 \quad x = 7$ A1




B1

B1 $5 \leq x \leq 7$

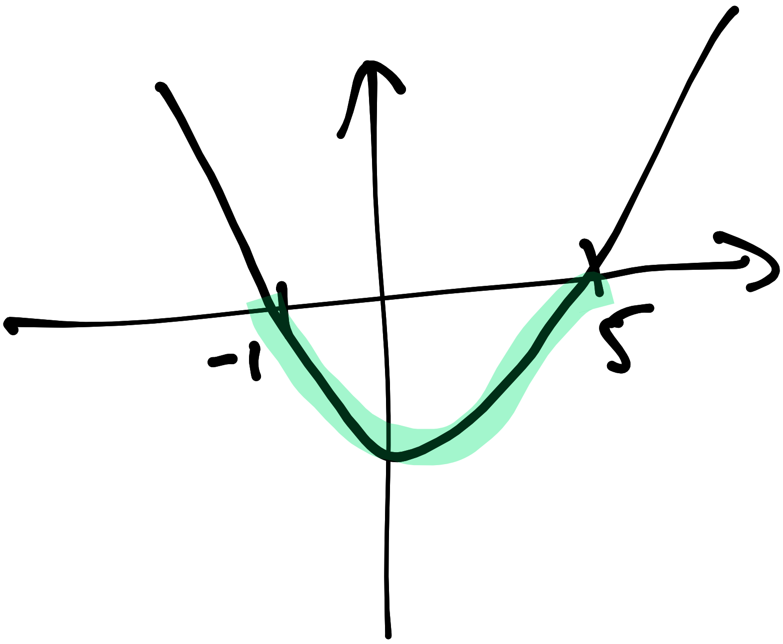
**(4 marks)**

**Q4.** AtoZrevision.com

Solve the inequality $x^2 - 4x - 5 < 0$

$(x+1)(x-5) < 0$ M1

$x = -1 \quad x = 5$ A1




B1 B1

$-1 < x < 5$

**(4 marks)**

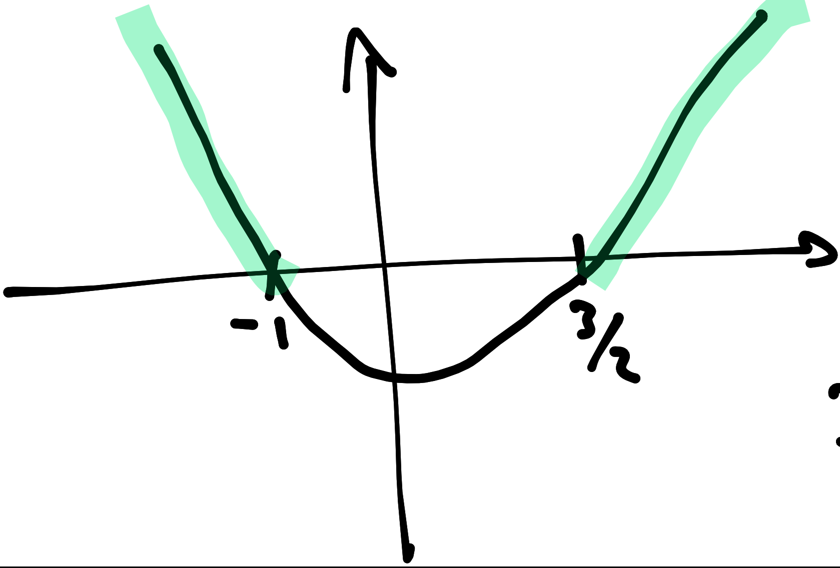
## Q5.



Solve the inequality $2x^2 - x - 3 > 0$

$(2x-3)(x+1) > 0$ M1

$x = \frac{3}{2}$ $x = -1$ A1




B1 B1

$x < -1$ $x > 3/2$

**(4 marks)**

## Q6.

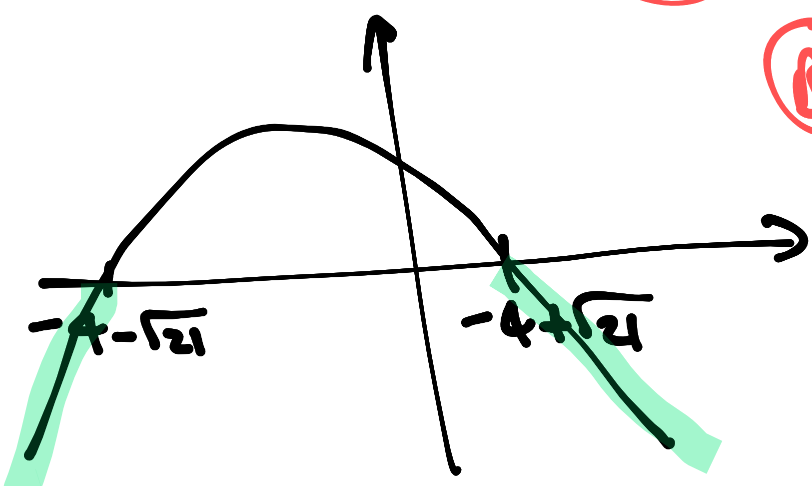


Solve the inequality $5 - 8x - x^2 \leq 0$, giving your answers in simplified surd form.

$a = -1$ $b = -8$ $c = 5$

$x = \dfrac{8 \pm \sqrt{64 - (4 \times -1 \times 5)}}{2 \times -1}$ M1

$= -4 \pm \sqrt{21}$ A1




B1 $x \geq -4 + \sqrt{21}$

$x \leq -4 - \sqrt{21}$ B1

**(4 marks)**

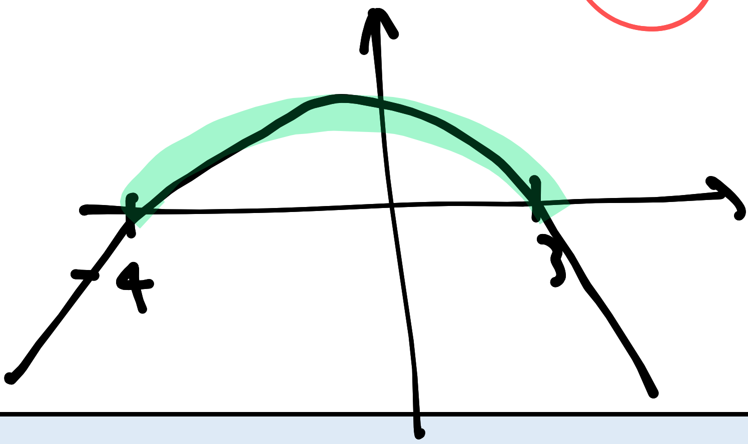
**Q7.** AtoZrevision.com

Solve the inequality $12 - x - x^2 > 0$

$a = -1 \quad b = -1 \quad c = 12$

$$x = \frac{1 \pm \sqrt{1 - (4 \times -1 \times 12)}}{2 \times -1}$$ M1

$x = 3 \quad x = -4$ A1




B1 B1

$-4 < x < 3$

**(4 marks)**

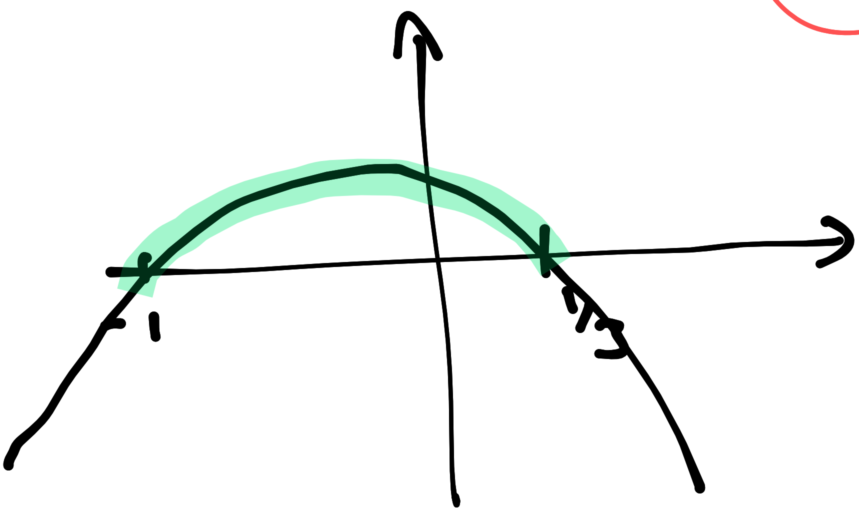
**Q8.** AtoZrevision.com

Solve $1 - 2x - 3x^2 > 0$, where $x$ is a real number.

$a = -3 \quad b = -2 \quad c = 1$

$$x = \frac{2 \pm \sqrt{4 - (4 \times -3 \times 1)}}{2 \times -3}$$ M1

$x = -1 \quad x = \frac{1}{3}$ A1




B1 B1

$-1 < x < \frac{1}{3}$

**(4 marks)**

**Q9.** AtoZrevision.com

The set of values of $x$ for which $3 + 2x \leq x + 2$ are $x \leq -1$

The set of values of $x$ for which $8x^2 + 10x < 3$ are $-\frac{3}{2} < x < \frac{1}{4}$

Hence, find the set of values of $x$ for which **both** $3 + 2x \leq x + 2$ **and** $8x^2 + 10x < 3$.

$-3/2 \leq x < \frac{1}{4}$

B1 B1

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**(2 marks)**

**Q10.** AtoZrevision.com

The set of values of $x$ for which $3(2x + 1) > 5 - 2x$ is $x > \frac{1}{4}$

The set of values of $x$ for which $2x^2 - 7x + 3 > 0$ is $x < \frac{1}{2}$ or $x > 3$.

Find the set of values of $x$ for which **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

$1/4 < x < 1/2$ B1

$x > 3$ B1

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**(2 marks)**

**Q11.** 

Find the set of values for $x$ for which both

$$8x - 7 < 5x + 5 \quad \text{and} \quad 2x^2 - 5x - 3 > 0$$

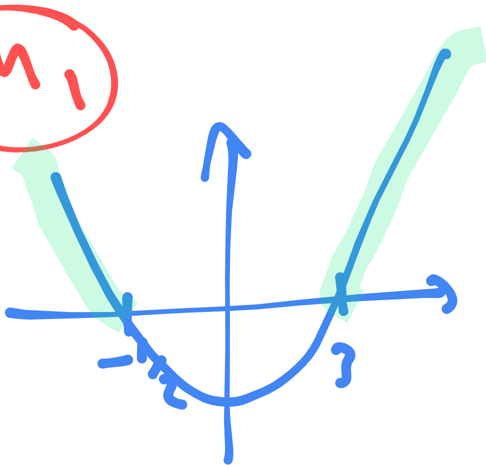
$3x < 12$

$x < 4$

M1

M1 $(2x+1)(x-3) > 0$

$x = -\frac{1}{2} \quad x = 3$ M1




$x < -\frac{1}{2}$

$x > 3$

A1

$\therefore \; x < -\frac{1}{2}$

$3 < x < 4$ B1

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**(5 marks)**

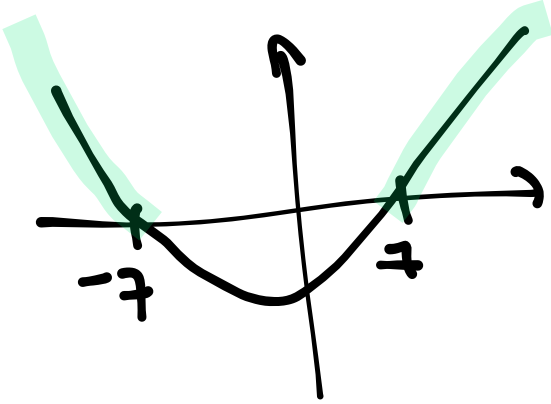
Find, algebraically, the set of values of $x$ for which

$$x^2 - 49 > 0 \quad \textbf{and} \quad 5x^2 - 31x - 72 > 0$$

$(x+7)(x-7) > 0$

M1

$x = 7 \quad x = -7$




$x < -7$

$x > 7$

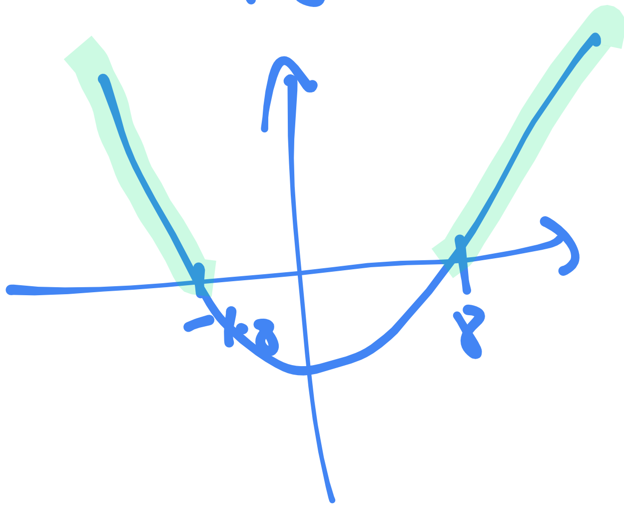
A1

$(5x+9)(x-8) > 0$

M1

$x = \frac{-9}{5} \quad x = 8$

$= -1.8$




$x < 1.8$

$x > 8$

A1

$\therefore \quad x < 7$

$x > 8$

A1

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**(5 marks)**

$$a^2 < 4 \quad \text{and} \quad a + 2b = 8$$

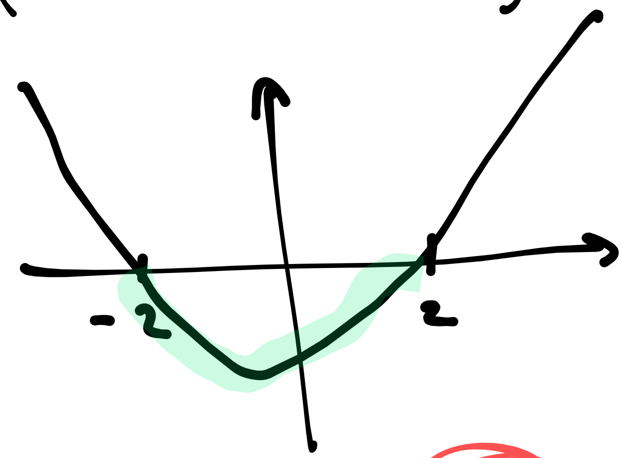
Work out the range of possible values of b.

Give your answer as an inequality.

1

$a^2 - 4 < 0$

$(a+2)(a-2) < 0$




$-2 < a < 2$ M1

2

$2 + 2b = 8$

$b = 3$ M1

$-2 + 2b = 8$

$b = 5$ M1

3 $3 < b < 5$ A1

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**(4 marks)**

$n$ is an integer such that $3n + 2 \leq 14$ and $\frac{6n}{n^2+5} > 1$

Find all the possible values of $n$.

$3n \leq 12$

$n \leq 4$ A1

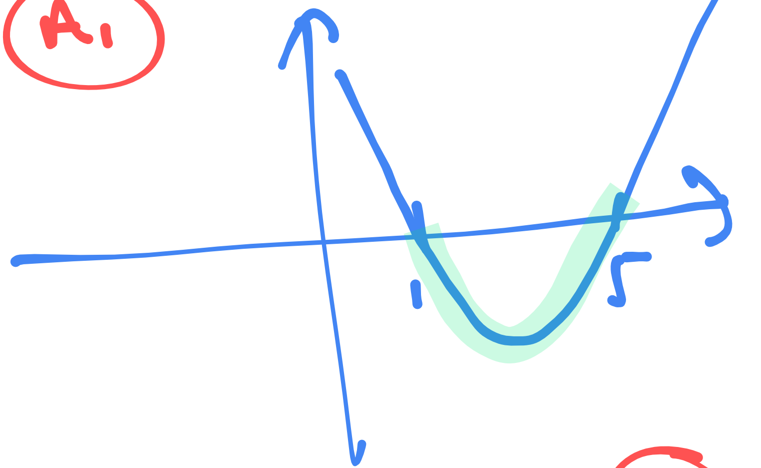
$6n > n^2 + 5$

M1 $0 > n^2 - 6n + 5$

$(n-1)(n-5) < 0$

$n = 1 \quad n = 5$

A1




$1 < n < 5$ A1

$\therefore \quad n = 2, 3, 4$ A1

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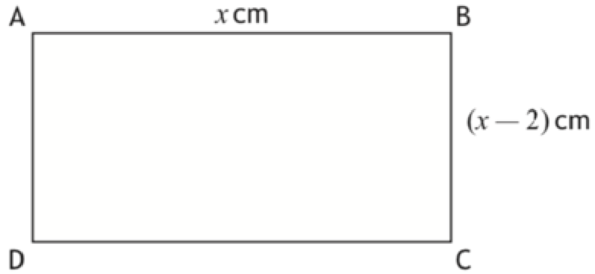
**(5 marks)**

**Q15.** 

ABCD is a rectangle with sides of lengths $x$ centimetres and $(x - 2)$ centimetres, as shown below.




If the area of ABCD is less than 15 cm$^2$, determine the range of possible values of $x$.

$x(x-2) < 15$ M1

$x^2 - 2x - 15 < 0$

$(x-5)(x+3) < 0$ M1

A1 $x = 5$ $\cancel{x = -3}$ NOT POSSIBLE

$2 < x < 5$ A1

HEIGHT WOULD BE 0.

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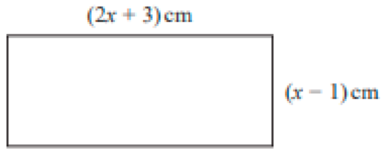
**(4 marks)**

Here is a rectangle.




Given that the area of the rectangle is less than 75 cm [2]
find the range of possible values of $x$

$(2x+3)(x-1) < 75$ M1

$2x^2 - 2x + 3x \cancel{- 3 - 75} < 0$

$-78$

$2x^2 + x - 78 < 0$ A1

$(2x+13)(x-6) < 0$ M1

$\cancel{x = \frac{-13}{2}}$ $x = 6$ A1

NOT POSSIBLE

$\therefore \quad 1 < x < 6$ A1

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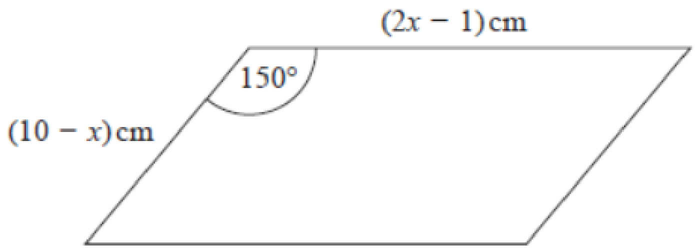
**(5 marks)**

**Q17.** 

The diagram shows a parallelogram.




The area of the parallelogram is greater than 15 cm $^2$.
It can be shown that $2x^2 - 21x + 40 < 0$.

Find the range of possible values of $x$.

$\frac{1}{2}(10-x)(2x-1) \times \sin 150 > 15$ M1

$2x^2 - 21x + 40 < 0$

$(2x-5)(x-8) < 0$ M1

$x = 5/2 \quad x = 8$ A1

$\therefore \quad 2.5 < x < 8$ A1

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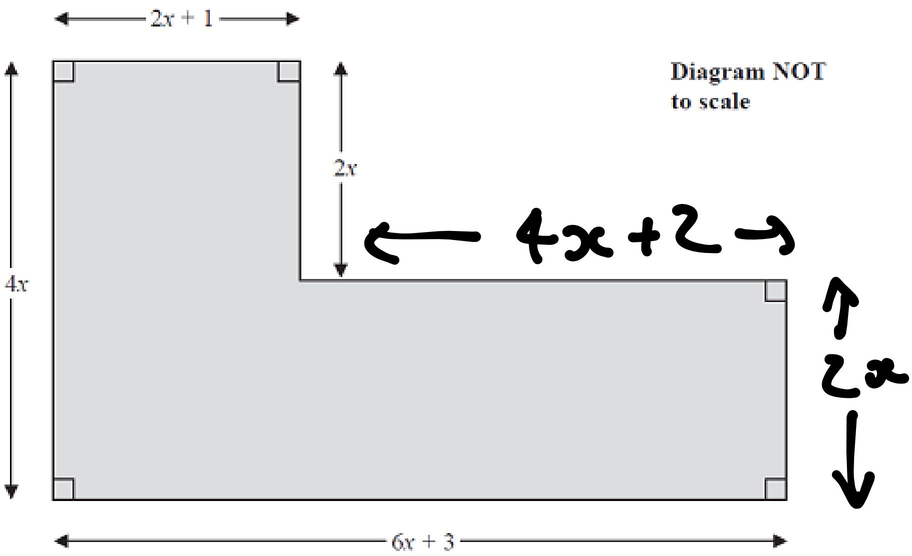
**(3 marks)**

**Q18.** 

Figure 1 shows the plan of a garden. The marked angles are right angles.
The six edges are straight lines. The lengths shown in the diagram are given in metres.




Given that the perimeter of the garden is greater than 40 m, it can be shown that the set of values for $x$ is $x > k$. Determine the value of $k$.

$P = 20x + 6$ M1

$20x + 6 > 40$ M1

$x > 1.7$ A1

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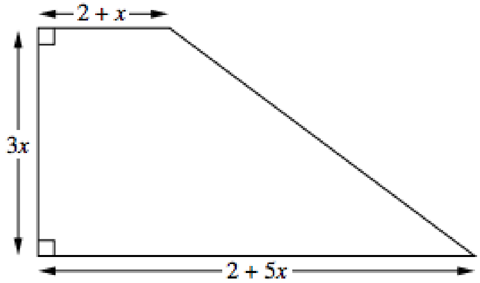
**(4 marks)**

A lawn is to be made in the shape shown below. The units are metres.




The perimeter of the lawn is $P = 14x + 4$ and the area is $A = 9x^2 + 6x$

The perimeter of the lawn must be at least 39 m and the area of the lawn must be less than 99 $m^2$.

By writing down and solving appropriate inequalities, determine the set of possible values of $x$.

B1 $14x + 4 \geq 39$

B1 $x \geq 2.5$

M1 $9x^2 + 6x < 99$

$3x^2 + 2x - 33 < 0$

$(3x + 11)(x - 3) < 0$

$\cancel{x = -\frac{11}{3}}$ $x = 3$ A1

$\therefore\ 2.5 \leq x < 3$ A1 A1

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**(6 marks)**

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than 180 m $^2$.

The width of the flower bed is $x$ m.

By writing down and solving appropriate inequalities in $x$, determine the set of possible values for the width of the flower bed.

$x + 3 \geq 14.5$ M1

$x \geq 11.5$ A1

$x(x+3) < 180$ M1

$x^2 + 3x - 180 < 0$

$(x-12)(x+15) < 0$ M1

$x = 12 \quad x = -15$ A1

$\therefore \quad 11.5 \leq x < 12$ B1

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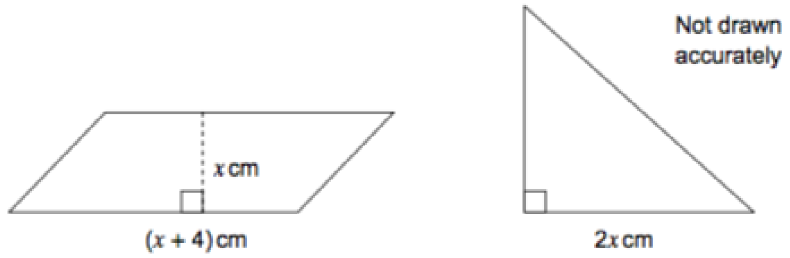
**(6 marks)**

**Q21.** 

Here are a parallelogram and an isosceles triangle.




The area of the triangle is greater than the area of the parallelogram.

Work out the least integer value for $x$.

$x(x+4) < \frac{1}{2} \times 2x \times 2x$ M1

$x^2 + 4x < 2x^2$

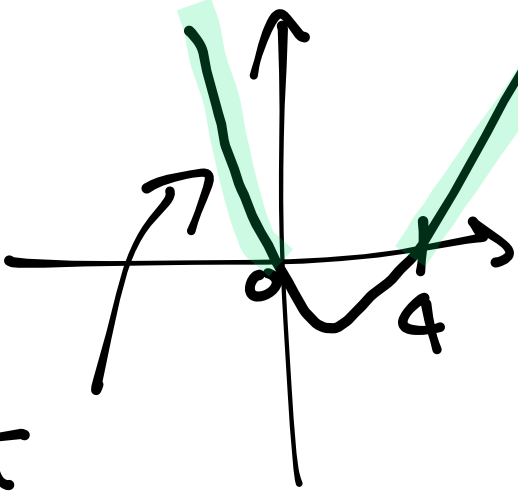
$x^2 - 4x > 0$ M1

$x(x-4) > 0$ M1

$x = 0$

$x = 4$

A1




NOT POSSIBLE

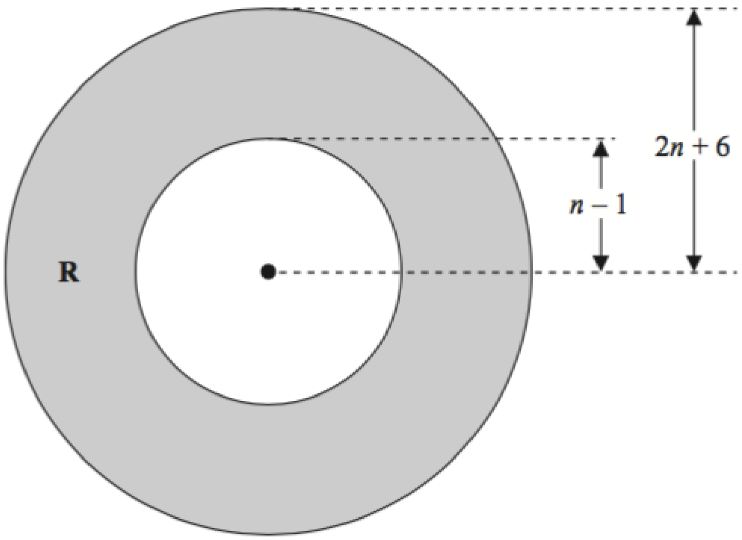
M1

FIRST INTEGER $>4$ IS 5

$x = 5$ A1 ……………………………

**(6 marks)**

The region **R**, shown shaded in the diagram, is the region between two circles with the same centre.




The outer circle has radius $(2n + 6)$
The inner circle has radius $(n - 1)$
All measurements are in centimetres.

The area of **R** is greater than the area of a circle of radius $(n + 13)$ cm. $n$ is an integer.

Find the least possible value of $n$.

P1 $\pi(2n+6)^2 - \pi(n-1) > \pi(n+13)^2$

P1

P1 $4n^2+24n+36 - n^2+2n-1 > n^2+26n+169$

$n^2 > 67$ P1

LEAST INTEGER WHOSE SQUARE > 67

$\therefore \underline{\underline{9}}$ A1

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**(5 marks)**

# QUESTIONS FROM MATHEMATICAL COMPETITIONS

**Q1.** AtoZrevision.com

An integer $x$ satisfies the inequality $x^2 \leq 729 \leq -x^3$. $P$ and $Q$ are possible values of $x$.

What is the maximum possible value of $10(P - Q)$?

$$x^2 \leq 729$$

$$\therefore \quad -27 \leq x \leq 27$$

$$729 \leq -x^3$$

$$x^3 \leq 729$$

$$x \leq -9$$

$$\therefore \quad -27 \leq x \leq -9$$

$\therefore$ MAX P−Q IS $-9-(-27)=18$

$$10(18) = \underline{180}$$