QUADRATIC SIMULTANEOUS EQUATIONS

GRADE 8

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

Solve the simultaneous equations

$$y = x^2$$

$$y = 7x - 10$$

$x^2 = 7x - 10$ M1

$x^2 - 7x + 10 = 0$ M1

$(x - 5)(x - 2) = 0$ M1

$x = 5 \qquad x = 2$ A1

$y = 25 \qquad y = 4$ A1

x = ... y = ...

x = ... y = ...

(5 marks)

Solve the simultaneous equations

$$y = 2x^2 + 4x + 1$$

$$y = x + 3$$

$x + 3 = 2x^2 + 4x + 1$ M1

$2x^2 + 3x - 2 = 0$ M1 A1

$(2x - 1)(x + 2) = 0$ M1

$x = \frac{1}{2}$ $\quad x = -2$ A1

$y = 3\frac{1}{2}$ $\quad y = 1$ A1

$x = \ldots\ldots\ldots\ldots\ldots\ldots$ $y = \ldots\ldots\ldots\ldots\ldots\ldots$

$x = \ldots\ldots\ldots\ldots\ldots\ldots$ $y = \ldots\ldots\ldots\ldots\ldots\ldots$

(6 marks)

Solve the simultaneous equations

$$x^2+y^2=5$$

$$y=3x+1$$

$$x^2+(3x+1)^2=5$$ M1

$$x^2+9x^2+6x+1=5$$ M1

$$10x^2+6x-4=0$$

$\div 2$ $$5x^2+3x-2=0$$ A1

$$(5x-2)(x+1)=0$$ M1

$x=4/5$ $\quad x=-1$ A1

$y=2.2$ $\quad y=-2$ A1

x = ... y = ...

x = ... y = ...

(6 marks)

Solve the simultaneous equations

$$y = 2x - 3$$

$$x^2 + y^2 = 2$$

$x^2 + (2x-3)^2 = 2$ M1

$x^2 + 4x^2 - 12x + 9 = 2$ M1

$5x^2 - 12x + 7 = 0$ A1

$(5x-7)(x-1) = 0$

$x = 7/5 \quad x = 1$ A1

$y = -1/5 \quad y = -1$ A1

x = ... y = ...

x = ... y = ...

(5 marks)

Solve the simultaneous equations

$$3x^2+y^2-xy=5$$
$$y=2x-3$$

M1

$$3x^2+(2x-3)^2-x(2x-3)=5$$

$$3x^2+4x^2-12x+9-2x^2+3x=5$$

M1

$$5x^2-9x+4=0$$

A1

$$x=\frac{9\pm\sqrt{81-(4\times5\times4)}}{2\times5}$$

$x=0.8$ $\quad$ $x=1$ A1

$y=-1.4$ $\quad$ $y=-1$ A1

(5 marks)

Q6. AtoZrevision.com

Solve the simultaneous equations

$$3xy - y^2 = 8$$

$$x - 2y = 1 \quad \therefore \quad x = 2y + 1$$

$$3(2y+1)y - y^2 = 8 \quad M1$$

$$6y^2 + 3y - y^2 = 8 \quad M1$$

$$5y^2 + 3y - 8 = 0 \quad A1$$

$$(5y+8)(y-1) = 0$$

$$y = -8/5 \qquad y = 1 \quad A1$$

$$x = -11/5 \qquad x = 3 \quad A1$$

(5 marks)

Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$

$$x + 2y - 1 = 0 \quad \therefore \; x = 1 - 2y$$ M1

$$(1-2y)^2 - 9y - (1-2y) = 2y^2 - 12$$

$$1 - \cancel{4y} + 4y^2 \cancel{-9y} - 1 \cancel{+2y} = 2y^2 - 12$$ M1

$$2y^2 - 11y + 12 = 0$$ A1

$$(2y-3)(y-4) = 0$$

$y = 3/2$ $y = 4$ A1

$x = -2$ $x = -7$ A1

(5 marks)

The straight line L has equation $x - y = 3$ $\therefore$ $x = y + 3$

The curve C has equation $3x^2 - y^2 + xy = 9$

L and C intersect at the points P and Q.

Find the coordinates of the midpoint of PQ.
Show clear algebraic working.

$$3(y+3)^2 - y^2 + (y+3)y = 9 \quad \text{M1}$$

$$3y^2 + 18y + 27 \cancel{-y^2} + \cancel{y^2} + 3y - 9 = 0$$

$$3y^2 + 21y + 18 = 0 \quad \text{M1}$$

$\div 3$

$$y^2 + 7y + 6 = 0 \quad \text{A1}$$

$$(y+1)(y+6) = 0 \quad \text{M1}$$

$y = -1$ $\quad$ $y = -6$ $\quad$ A1

$x = -3$ $\quad$ $x = 2$

MIDPOINT: $\left(-\frac{1}{2}, -\frac{7}{2}\right)$ A1

(6 marks)

The curve with equation $x^2 - x + y^2 = 10$ and the straight line with equation $x - y = -4$ intersect at the points A and B.

$\therefore x = y - 4$

Work out the exact length of AB.

Give your answer in the form $\frac{\sqrt{a}}{2}$ where a is an integer.

$$(y-4)^2 - (y-4) + y^2 = 10 \quad M1$$

$$y^2 - 8y + 16 - y + 4 + y^2 = 10$$

$$2y^2 - 9y + 10 = 0 \quad A1$$

$$(2y-5)(y-2) = 0$$

$$y = 5/2 \qquad y = 2 \quad A1$$

$$x = -1/2 \qquad x = -2 \quad A1$$

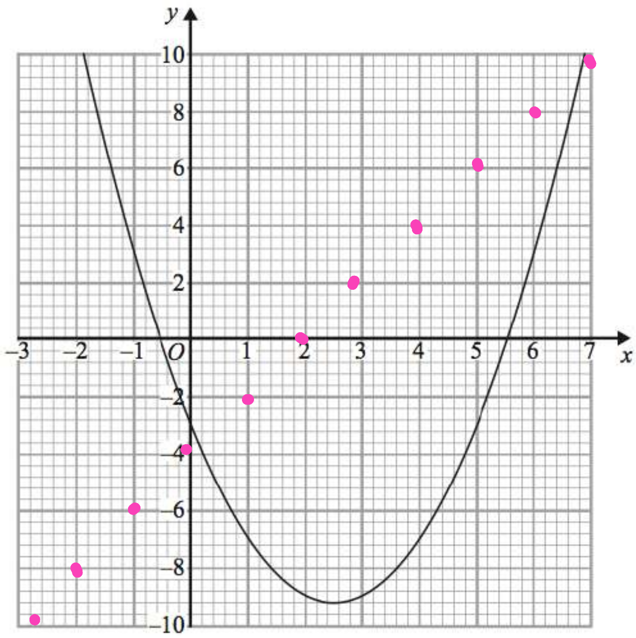
$$\sqrt{(-1/2 - -2)^2 + (2.5 - 2)^2} \quad M1$$

$$= \frac{\sqrt{2}}{2} \quad A1$$

(6 marks)

Q10.

The diagram shows the graph of $y = x^2 - 5x - 3$.



$y = x - 4$

± 0.1

Use the graph to find estimates for the solutions of the simultaneous equations

$$y = x^2 - 5x - 3$$
$$y = x - 4$$

$x = 0.2 \quad y = -3.8$ A1

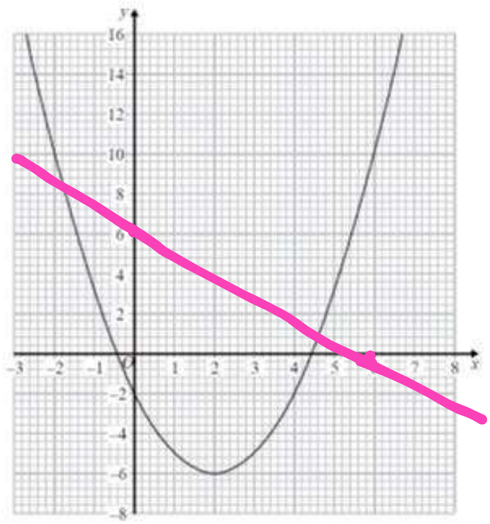
$x = 5.8 \quad y = 1.8$ A1

(2 marks)

Q11.

The diagram shows the graph of $y = x^2 - 4x - 2$

(NOT ± 0.1)
± 0.2



Use the graph to find estimates for the values of *x* that satisfy the simultaneous equations

$$y = x^2 - 4x - 2$$
$$x + y = 6$$

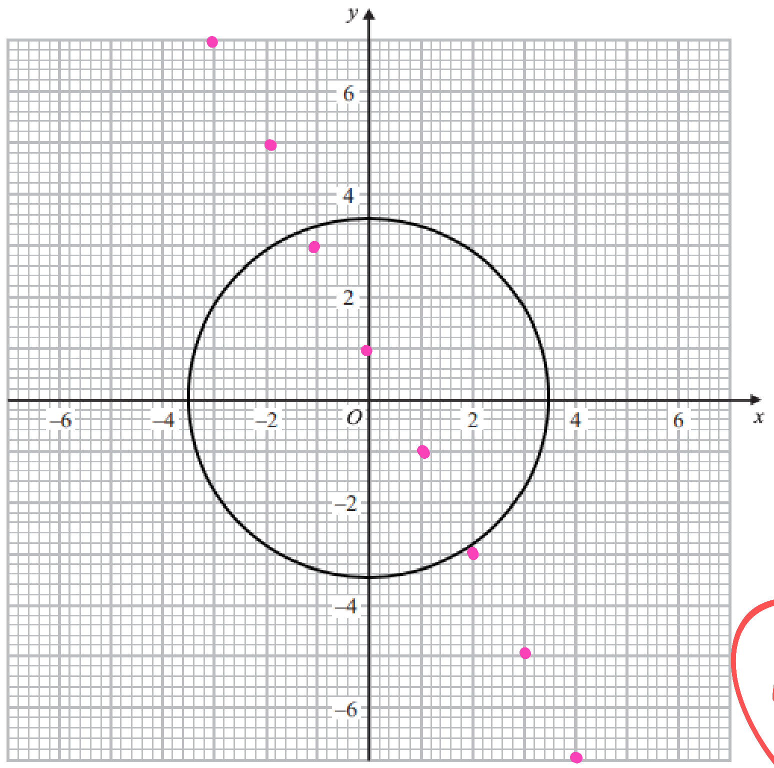
$x = 4.6$ A1

$x = -1.8$ A1

(2 marks)

Q12. AtoZrevision.com

The graph of $x^2+y^2 = 12.25$ is drawn below.



M1

Hence find estimates for the solutions of the simultaneous equations $y = -2x + 1$

A1

$$x^2+y^2 = 12.25$$
$$2x + y = 1$$

$x = 2$ $\quad$ $x = -1.2$

$y = -2.9$ $\quad$ $y = 3.3$

A1

(3 marks)

Q13. AtoZrevision.com

A curve has the equation $y = 2x^2 - 8x - 5$.

$y = 0$

Find the x coordinates of the two points where the curve intercepts the x-axis.

$2x^2 - 8x - 5 = 0$

M1

USE QUADRATIC FORMULA

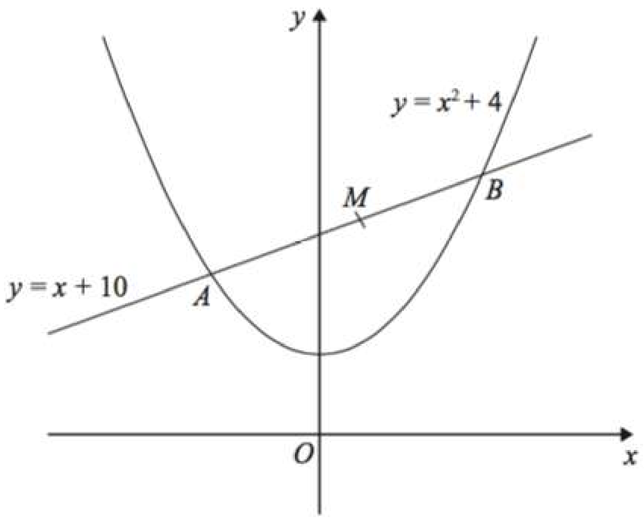
$2 \pm \sqrt{\frac{13}{2}}$ ....

A1

(2 marks)

Q14. AtoZrevision.com

The sketch shows the curve with equation $y = x^2 + 4$ and the line with equation $y = x + 10$



The line cuts the curve at the points A and B.

M is the midpoint of AB.

Find the coordinates of M.

$x^2 + 4 = x + 10$ M1

$x^2 - x - 6 = 0$ M1

$(x-3)(x+2) = 0$ M1

$x = 3 \quad x = -2$ A1

$y = 13 \quad y = 8$ A1

MIDPOINT $= (0.5, 10.5)$ A1

……………………………

(6 marks)

Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

$x = 2y + 10$

$(2y+10)^2 + y^2 = 20$ M1

$4y^2 + 40y + 100 + y^2 = 20$ M1

$5y^2 + 40y + 80 = 0$ A1

SOLVES FOR:

$x = 2$ A1 $y = -4$ (1 SOLUTION)

THERE IS ONLY ONE POINT OF INTERSECTION $\therefore$ THE LINE IS A TANGENT. A1

(5 marks)

Q16. AtoZrevision.com

$a = 3^x$ and $b = 3^y$

$ab = 2187$
$a^2b = 177147$

Work out the value of x and the value of y.

$3^x = 81$ $\therefore x = 4$

$3^y = 27$ $\therefore y = 3$ A1

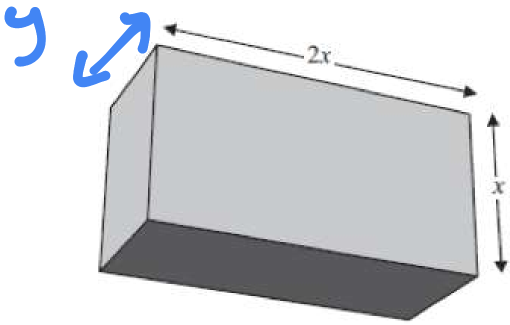
$a^2b = 177147$
$\div$ $ab = 2187$
$a = 81$ $\therefore b = 27$

M1 A1

....

(3 marks)

Q17. AtoZrevision.com



A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm.

The volume of the cuboid is 81 cubic centimetres. Show that the total length, L cm, of the twelve edges of the cuboid is given by $L = ax + \frac{b}{x^2}$

were a and b are integers to be found.

$(x)(2x)(y) = 81$ $\therefore 2x^2y = 81$

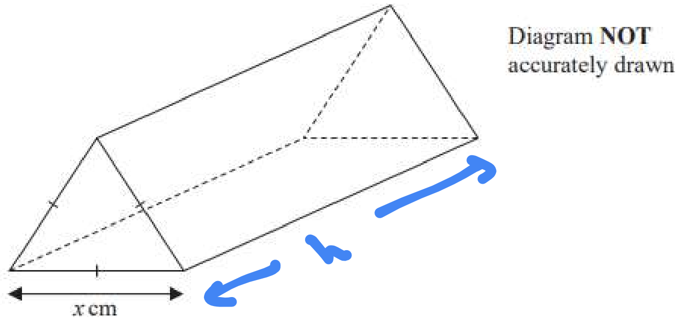
$4(2x) + 4(x) + 4(y) = L$

$L = 12x + 4y$ $\quad y = \frac{81}{2x^2}$ M1

M1 $= 12(x) + 4\left(\frac{81}{2x^2}\right)$

$= 12x + 162/x^2$ A1

(3 marks)



A company manufactures chocolate bars that are inside packaging that is in the shape of a right triangular prism.

The cross section of the prism is an equilateral triangle with sides of length x cm.

The volume of the prism is 72 cm ^{3}

The total surface area of the prism is S cm ^{2}

Show that S = $\frac{\sqrt{3}x^2}{m} + \frac{n\sqrt{3}}{x}$ where m and n are integers to be found.

$$\underbrace{\tfrac{1}{2}x^2\sin 60}_{\frac{\sqrt{3}}{2}}\, h = 72 \quad M_1$$

$$\frac{\sqrt{3}\,x^2 h}{4} = 72 \quad M_1$$

$$\therefore h = \frac{288}{\sqrt{3}\,x^2} \quad A_1$$

$$S = 2 \times \tfrac{1}{2}x^2 \sin 60 + 3xh \quad M_1$$

$$= \frac{\sqrt{3}\,x^2}{2} + 3x\left(\frac{288}{\sqrt{3}\,x^2}\right) \quad M_1$$

$$= \frac{\sqrt{3}\,x^2}{2} + \frac{288\sqrt{3}}{x} \quad A_1$$

(6 marks)

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

The positive numbers x and y satisfy the equations $x^4 - y^4 = 2009$ and $x^2 + y^2 = 49$.

What is the value of y?

$$(x^2+y^2)(x^2-y^2) = 2009$$

$$(49)(x^2-y^2) = 2009$$

$$x^2-y^2 = \frac{2009}{49} = 41$$

$$x^2+y^2 = 49$$

$$-\quad x^2-y^2 = 41$$

$$2y^2 = 8$$

$$y = 2 \text{ or } \cancel{-2}$$

Q2. AtoZrevision.com

$xy = 15$ and $(2x - y)^4 = 1$

where x and y are real numbers. She calculates z, the sum of the squares of all the y-values in her solutions.

What is the value of z?

$$(2x-y)^2 = \pm 1$$

TWO POSSIBLE ANSWERS.

A SQUARED VALUE ≠ −VE

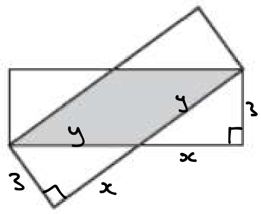
$$\therefore (2x-y)^2 = 1 \quad \therefore \quad 2x-y = \pm 1$$

$$(\times y) \quad 2xy - y^2 = y \qquad xy = 15 \quad \therefore \quad y = -6, 5$$

$$(\times -y) \quad 2x - y^2 = -y \qquad xy = 15 \quad \therefore \quad y = -5, 6$$

$$\therefore z = (-6)^2 + 5^2 + (-5)^2 + 6^2 = 122$$

Two identical rectangles with sides of length 3 cm and 9 cm are overlapping as in the diagram.



What is the area of the overlap of the two rectangles?

TRIANGLES ARE CONGRUENT

$$x^2 + 3^2 = y^2$$

$AD = 9 \therefore x + y = 9$

$$x = 9 - y$$

$(9-y)^2 + 3^2 = y^2 \quad \therefore 18y = 90$

$\therefore y = 5$

$x + 5 = 9 \quad \therefore x = 4$

OVERLAP:

$$3 \times 9 - 2 \times 6 = 15\text{cm}^2$$