ALGEBRAIC TREE DIAGRAMS

GRADE 9

AtoZrevision

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don’t forget to have fun.
- Check your answers at the end.

There are only n red balls and $(n+1)$ blue balls in a bag.

Shamsa takes at random 2 balls from the bag.

Find an expression for the probability that both balls are the same colour.

$\frac{n}{2n+1} \times \frac{n-1}{2n}$ TWO RED C1

$\frac{n+1}{2n+1} \times \frac{n}{2n}$ TWO BLUE C1

$\underbrace{\frac{n(n-1)}{2n(2n+1)} + \frac{n(n+1)}{2n(2n+1)}}_{C1} = \frac{n}{2n+1}$ C1

....

(4 marks)

A bag contains n beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, calculate the value of n.

M1

$$\left(\frac{6}{n} \times \frac{5}{n-1}\right) + \left(\frac{n-6}{n} \times \frac{n-7}{n-1}\right)$$

M1

$$= \frac{9}{17}$$

M1

$$\left[\frac{30}{n^2-n} + \frac{n^2-13n+42}{n^2-n} = \frac{9}{17}\right]$$

SIMPLIFIES TO $2n^2 - 53n + 306 = 0$

A1

$$(2n-17)(n-18) = 0$$

M1

$$\cancel{n = \frac{17}{2}} \qquad \boxed{n = 18}$$

A1

NOT POSSIBLE TO HAVE $17/2$ BEADS

....

(6 marks)

A box contains marbles.

4 of the marbles are red.
The rest of the marbles are yellow.

Antonia takes at random a marble from the box and does not replace it.
Sergio then takes at random a marble from the box.

The probability that Antonia and Sergio both take a yellow marble is 0.7

Work out how many marbles were originally in the box.

$$\frac{x-4}{x} \times \frac{x-5}{x-1} = 0.7 \quad \text{M1}$$

M1

$$3x^2 - 83x + 200 = 0 \quad \text{A1}$$

$$(3x-8)(x-25) = 0 \quad \text{M1}$$

$$\cancel{x = \frac{8}{3}} \qquad x = 25 \quad \text{A1}$$

....

(5 marks)

A bag contains X counters.

There are only red counters and blue counters in the bag.

There are 4 more blue counters than red counters in the bag.

Finty takes at random 2 counters from the bag.

The probability that Finty takes 2 blue counters from the bag is $\frac{3}{8}$

Work out the value of X.

BLUE:

$$\frac{B}{2B-4} \times \frac{B-1}{2B-5} = \frac{3}{8} \quad M_1$$

M_1

$$8B(B-1) = 3(2B-4)(2B-5) \quad M_1$$

$$4B^2 - 46B + 60 = 0 \quad M_1$$

$$2B^2 - 23B + 30 = 0$$

$\cancel{B = 1.5}$ $\quad B = 10$

$\therefore R = 6$

16 A_1

(5 marks)

Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

B = BLACK

$$\underbrace{\frac{B}{2B-3}}\times\underbrace{\frac{B-3}{2B-4}}=\frac{9}{35}$$

M1

M1 EITHER

$$35(B)(B-3)=9(2B-3)(2B-4)$$

M1

$$35(B^2-3B)=9(4B^2-14B+12)$$

$$B^2-21B+108=0$$

$B=12$ $\quad B=9$

$\therefore R=9$ $\quad \therefore R=6$

25 or 15 A1

A1

....

(5 marks)

A bag contains 40 marbles. x of these are whilte.

One marble is taken at random and then, without replacement, a second marble is taken at random.

The probability that neither of these is white is $\frac{5}{12}$.

Work out the number of white marbles in the bag.

$$\frac{40-x}{40} \times \frac{39-x}{39} = \frac{5}{12}$$ M1

M1 EITHER

$$1560 - 40x - 39x + x^2 = 40 \times 39 \times \frac{5}{12}$$

M1

$$x^2 - 79x + 1560 = 650$$

$$x^2 - 79x + 910 = 0$$ M1

$$(x-14)(x-65) = 0$$

$$x = 14$$

~~$x = 65$~~

A1

....

(6 marks)

Boris has a bag that only contains red sweets and green sweets.

Boris takes at random 2 sweets from the bag.

The probability that Boris takes exactly 1 red sweet from the bag is $\frac{12}{35}$.

Originally there were 3 red sweets in the bag.

Work out how many green sweets there were originally in the bag.

RG AND GR EITHER M1

$$\left(\frac{3}{t} \times \frac{t-3}{t-1}\right) + \left(\frac{t-3}{t} \times \frac{3}{t-1}\right) = \frac{12}{35}$$ M1

$$\frac{3t-9}{t^2-t} + \frac{3t-9}{t^2-t} = \frac{12}{35}$$

$$2t^2 - 37t + 105 = 0$$ M1

$$(2t-7)(t-15) = 0$$ M1

$\cancel{t = \frac{7}{2}}$ $t = 15$

$$15 - 3$$

$$12$$ A1

....

(5 marks)

There are some red counters and some white counters in a bag.
At the start, 7 of the counters are red, the rest of the counters are white.

Alfie takes at random a counter from the bag.
He does not put the counter back in the bag.
Alfie then takes at random another counter from the bag.

The probability that the first counter Alfie takes is white and the second counter Alfie takes is red is $\frac{21}{80}$.

Work out the number of white counters in the bag at the start.

$$\frac{7}{x+7} \times \frac{x}{x+6} = \frac{21}{80} \quad P1$$

$$\frac{7x}{x^2+13x+42} = \frac{21}{80} \quad P1$$

$$560x = 21(x^2+13x+42) \quad P1$$

$$21x^2 - 287x + 882 = 0 \quad P1$$

$$\cancel{x = \frac{14}{2}} \qquad \underline{\underline{x = 9}} \quad P1$$

....

(5 marks)

There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

P1 ONE CORRECT PROBABILITY

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.

P1 ONE CORRECT PRODUCT

$$\left(\frac{x}{2x+3} \times \frac{x-1}{2x+2}\right) +$$

$$\left(\frac{x+3}{2x+3} \times \frac{x+2}{2x+2}\right) = \frac{27}{55}$$ P1

$$\frac{x^2 - x}{(2x+3)(2x+2)} + \frac{x^2+5x+6}{(2x+3)(2x+2)} = \frac{27}{55}$$

$$x^2 - 25x + 84 = 0$$ P1

$$(x-21)(x-4) = 0$$ P1

$x = 21$ $\cancel{x = 4}$

P1

....

(6 marks)

A box contains 5 black bats and x red bats.

Ben takes 2 bats, without replacement, from the box.

The probability that both bats chosen are red is $\frac{1}{6}$

By forming an equation in x, find how many red bats are in the box.

$$\frac{x}{x+5} \times \frac{x-1}{x+4} = \frac{1}{6}$$

M1 M1

$$6x(x-1) = (x+5)(x+4)$$

M1

SIMPLIFYING

$$6x^2 - 6x = x^2 + 9x + 20$$

$$5x^2 - 15x - 20 = 0$$

$$x^2 - 3x - 4 = 0$$

A1

$$(x-4)(x+1) = 0$$

M1

$\cancel{x = -1}$ $\quad$ $\underline{\underline{x = 4}}$

A1

....

(6 marks)

A bowl contains n pieces of fruit.
Of these, 4 are oranges and the rest are apples.
Two pieces of fruit are going to be taken at random from the bowl.
The probability that the bowl will then contain $(n-6)$ apples is $\frac{1}{3}$

Work out the value of n.

$$\underbrace{\frac{n-4}{n}}_{\text{M1}} \times \underbrace{\frac{n-5}{n-1}}_{\text{EITHER}} = \frac{1}{3} \quad \text{M1}$$

$$\frac{n^2-9n+20}{n^2-n} = \frac{1}{3} \quad \text{M1}$$

$$3(n^2-9n+20) = n^2-n \quad \text{M1}$$

$$3n^2-27n+60 = n^2-n$$

$$2n^2-26n+60 = 0 \quad \text{M1}$$

$$n^2-13n+30 = 0 \quad \text{A1}$$

$$(n-10)(n-3) = 0 \quad \therefore \quad n = 10$$

....

(6 marks)

Q12.

There are only red counters, yellow counters and blue counters in a bag.

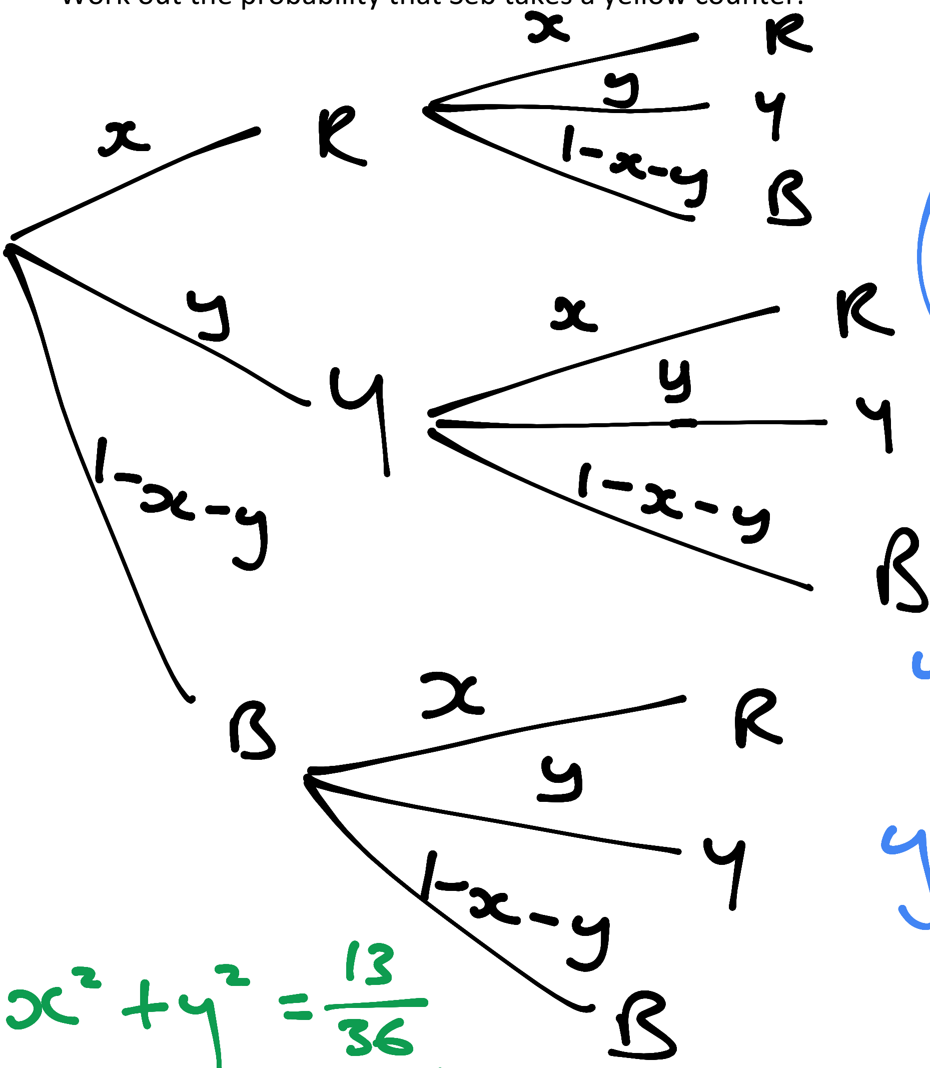
Kevin takes at random a counter from the bag.
He puts the counter back in the bag.
Lethna takes at random a counter from the bag.
She puts the counter back in the bag.

The probability that both counters are red or that both counters are yellow is $\frac{13}{36}$

The probability that the first counter is red and the second counter is not red is $\frac{1}{4}$

Seb takes at random a counter from the bag.

Work out the probability that Seb takes a yellow counter.



$\left(\frac{1}{2}\right)^2 + y^2 = \frac{13}{36}$

$\frac{1}{4} + y^2 = \frac{13}{36}$

$y^2 = \frac{13}{36} - \frac{1}{4} = \frac{4}{36}$

$y = \frac{1}{3}$

$x^2 + y^2 = \frac{13}{36}$

$x(1-x) = \frac{1}{4}$

$x - x^2 = \frac{1}{4}$

$x^2 - x + \frac{1}{4} = 0$

$\left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) \quad \therefore x = \frac{1}{2}$

$P(Y) = \frac{1}{3}$ ....

(5 marks)

A bag contains red counters and blue counters and white counters.

There are n red counters.
There are 2 more blue counters than red counters.
The number of white counters is equal to the total number of red counters and blue counters.

Bob and Ann play a game.

Bob will take a counter at random from the bag.
He will record the colour and put the counter back in the bag.
Ann will then take a counter at random from the bag.
She will record its colour.
The probability that Bob's counter is red and Ann's counter is **not** red is $\frac{14}{81}$

Bob and Ann play the game with all 36 counters in the bag.

Calculate the probability that Bob and Ann will take counters with different colours.

M1

$$\frac{n}{4(n+1)} \times \frac{3n+4}{4(n+1)} = \frac{14}{81}$$

M1

$$81n(3n+4) = 14 \times 16(n+1)^2$$

$$243n^2 + 324n = 224(n^2+2n+1)$$

$$19n^2 - 124n - 224 = 0$$

A1

$n = 8$ ~~$n = \frac{-28}{19}$~~

8 RED
10 BLUE
18 WHITE

A1

M1

$P(\text{SAME}) = RR + BB + WW$

$$\left(\frac{8}{36}\right)^2 + \left(\frac{10}{36}\right)^2 + \left(\frac{18}{36}\right)^2$$

$$= \frac{61}{162}$$

$$1 - \frac{61}{162} = {}^{101}/_{162}$$

A1

PHEW!

(6 marks)

Here are 9 cards.
A bag only contains red marbles, blue marbles and yellow marbles.

- The probability of picking a red marble is $\frac{2}{5}$.
- There are nine yellow marbles. x
- The probability of picking a blue marble is three times as likely as picking a yellow marble. $3x$

M1

Work out the total number of marbles in the bag.

$$\frac{2}{5} + x + 3x = 1$$ M1

M1

$$4x = \frac{3}{5}$$

$$x = 3/20$$ A1

$$\frac{3t}{20} = 9$$

$$t = \frac{9 \times 20}{3} = 60$$ A1

....

(5 marks)

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1. AtoZrevision.com

There are 10 girls in a mixed class. If two pupils from the class are selected at random to represent the class on the School Council, then the probability that both are girls is 0.15.

How many boys are there in the class?

$$\frac{10}{10+x} \times \frac{9}{9+x} = \frac{90}{(10+x)(9+x)} = 0.15$$

SIMPLIFY: $x^2 + 19x - 510 = 0$

$$(x+34)(x-15) = 0$$

$$\cancel{x = -34} \quad \underline{\underline{x = 15}}$$

Q2. AtoZrevision.com

A bag contains m blue and n yellow marbles. One marble is selected at random from the bag and its colour is noted. It is then returned to the bag along with k other marbles of the same colour. A second marble is now selected at random from the bag. What is the probability, in terms of m, n and k, that the second marble is blue?

1^{st} B $= \frac{m}{m+n}$ $\quad$ 1^{st} Y $\frac{n}{m+n}$

2^{nd} B $= \frac{m+k}{m+n+k}$ $\quad$ 2^{nd} B $\frac{m}{m+n+k}$

$$\therefore \frac{m}{m+n} \times \frac{m+k}{m+n+k} + \frac{n}{m+n} \times \frac{m}{m+n+k} =$$

$$\frac{m(m+k)+mn}{(m+n)(m+n+k)} = \frac{m\cancel{(m+n+k)}}{(m+n)\cancel{(m+n+k)}} = \frac{m}{m+n}$$

$\ddot{\smile}$

Tom and Geri have a competition. Initially, each player has one attempt at hitting a target. If one player hits the target and the other does not then the successful player wins. If both players hit the target, or if both players miss the target, then each has another attempt, with the same rules applying.

If the probability of Tom hitting the target is always $\frac{4}{5}$ and then probability of Geri hitting the target is always $\frac{2}{3}$, what is the probability that Tom wins the competition? $= W$

TOM HITS AND GERI MISSES:

$$\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

BOTH HIT:

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

BOTH MISS:

$$\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$$

$$\frac{8}{15} + \frac{1}{15} = \frac{3}{5}$$

COMPETITION IS EQUAL ∴

$$W = \frac{4}{15} + \frac{3}{5}W$$

$$\frac{2}{5}W = \frac{4}{15}$$

$$\therefore W = \frac{2}{3}$$