

CIRCLE THEOREMS (HIGHER)

GRADE 6

Examination Practice Questions

You should have:

A ruler, protractor, compasses, a pen, pencil, eraser, calculator.
For some questions, you may need tracing paper.

Instructions

- Use **black** ink or ball-point pen.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets.
- Use the number of marks for each question as a guide as to how much time to spend on each question. As a rough guide, you can multiply the number of marks by 1.2 to see how many minutes you should spend on a question.
- Questions been carefully compiled from or modelled on a variety of past papers and will generally get more challenging as the document progresses. Some of the later questions will go beyond the core grade level for this topic.

Advice

- Read each question carefully before you start to answer it.
- Don't forget to have fun.
- Check your answers at the end.

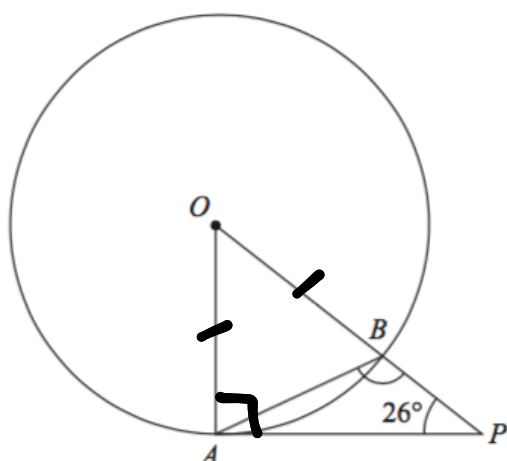


Diagram NOT
accurately drawn

A and B are points on a circle, centre O.

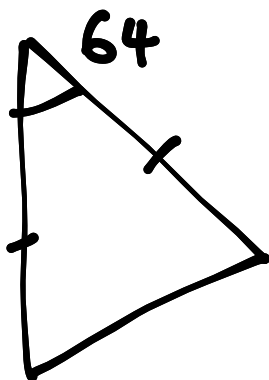
PA is the tangent to the circle at A.

OBP is a straight line.

Angle APO = 26° .

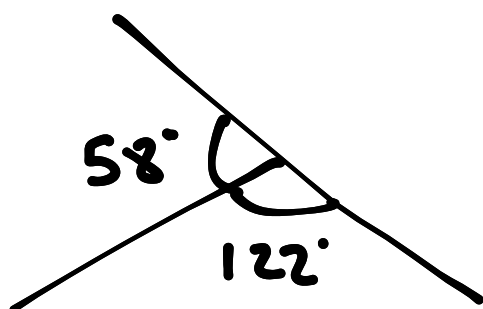
Calculate the size of angle ABP.

$$180 - 90 - 26 = \angle AOP = 64 \quad \text{M1}$$



$$\frac{180 - 64}{2}$$

$$\text{M1} = 58^\circ = \angle OBA$$



$$\text{A1} \quad 122^\circ$$

(3 marks)

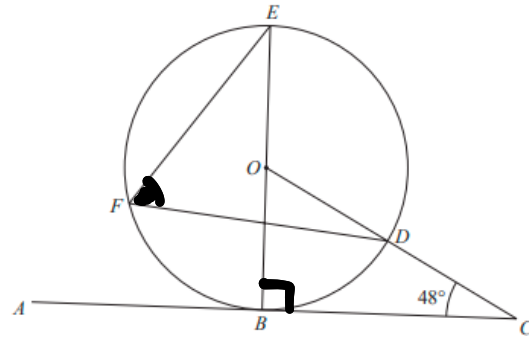


Diagram NOT
accurately drawn

B, D, E and F are points on a circle, centre O .

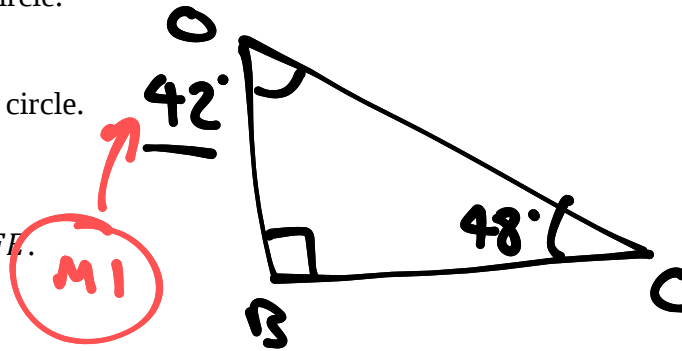
ABC is a tangent to the circle.

ODC is a straight line.

BOE is a diameter of the circle.

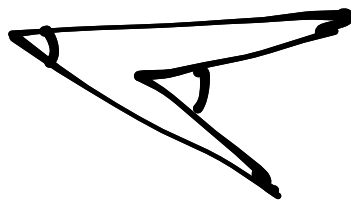
Angle $BCD = 48^\circ$

Find the size of angle DFE .



$$\begin{aligned}\angle EOD &= 180 - 42 \\ &= 138^\circ\end{aligned}$$

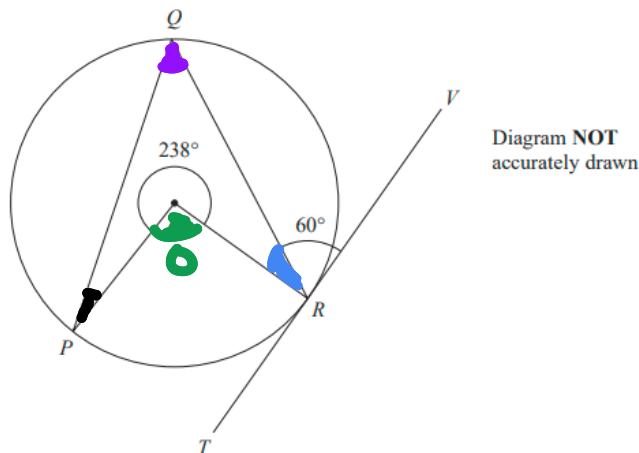
$$\angle DFE = \text{HALF } \angle EOD$$



$$\therefore \angle DFE = 69^\circ$$

.....
(3 marks)

P , Q and R are points on a circle, centre O . TRV is the tangent to the circle at R .



Reflex angle $POR = 238^\circ$

Angle $QRV = 60^\circ$

Calculate the size of angle OPQ .

$$\angle VRQ = 90^\circ \text{ (TANGENT - RADIUS)}$$

$$\angle ORQ = 90 - 60 = 30^\circ \quad \text{M1}$$

$$\begin{aligned} \angle POR &= 360 - 238 \\ &= 122^\circ \quad \text{M1} \end{aligned}$$

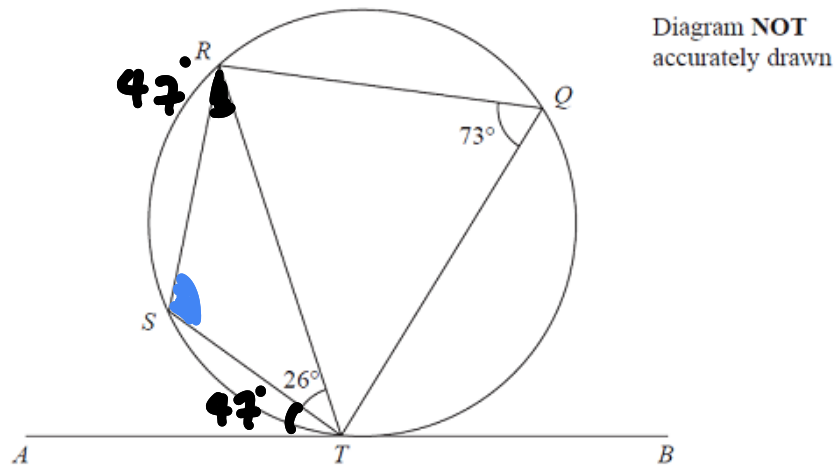
$$\angle PQR = \text{HALF OF } 122^\circ = 61^\circ \quad \text{M1}$$

$$\begin{aligned} \angle OPQ &= 360 - 30 - 61 - 238 \\ &= 31^\circ \quad \text{A1} \\ &= \underline{\underline{31}} \end{aligned}$$

Q, R, S and T are points on a circle.
 ATB is the tangent to the circle at T

$$\text{Angle } STR = 26^\circ$$

$$\text{Angle } RQT = 73^\circ$$



Work out the size of angle STA

$$\begin{aligned} \angle RST &= 180 - 73 \\ &= 107^\circ \quad (M1) \end{aligned}$$

$\angle SRT$:

$$180 - 107 - 26 = 47^\circ \quad (M1)$$

$$\therefore \angle STA = 47^\circ \quad (M1)$$

ALTERNATE
SEGMENT THEOREM

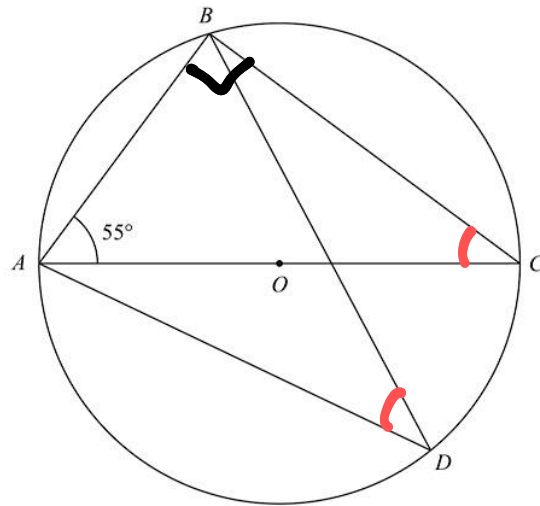


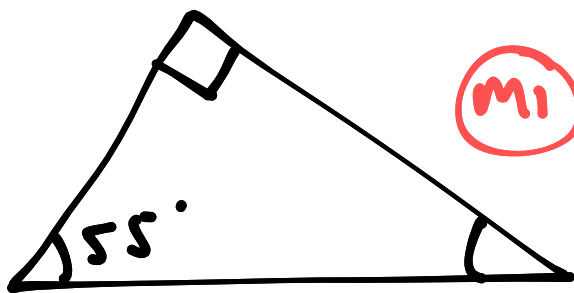
Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle, centre O
 AOC is a diameter of the circle.

Angle $BAC = 55^\circ$

Work out the size of angle ADB

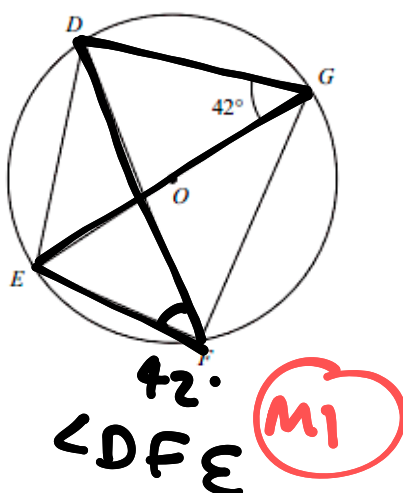
$$\angle ABC = 90^\circ \quad (M_1)$$



$$(M_1) \left[180 - 90 - 55 = 35^\circ \right. \\ \left. \angle BCA = 35^\circ \right] \quad (A_1)$$

$$\therefore \angle ADB = 35^\circ \quad (A_1)$$

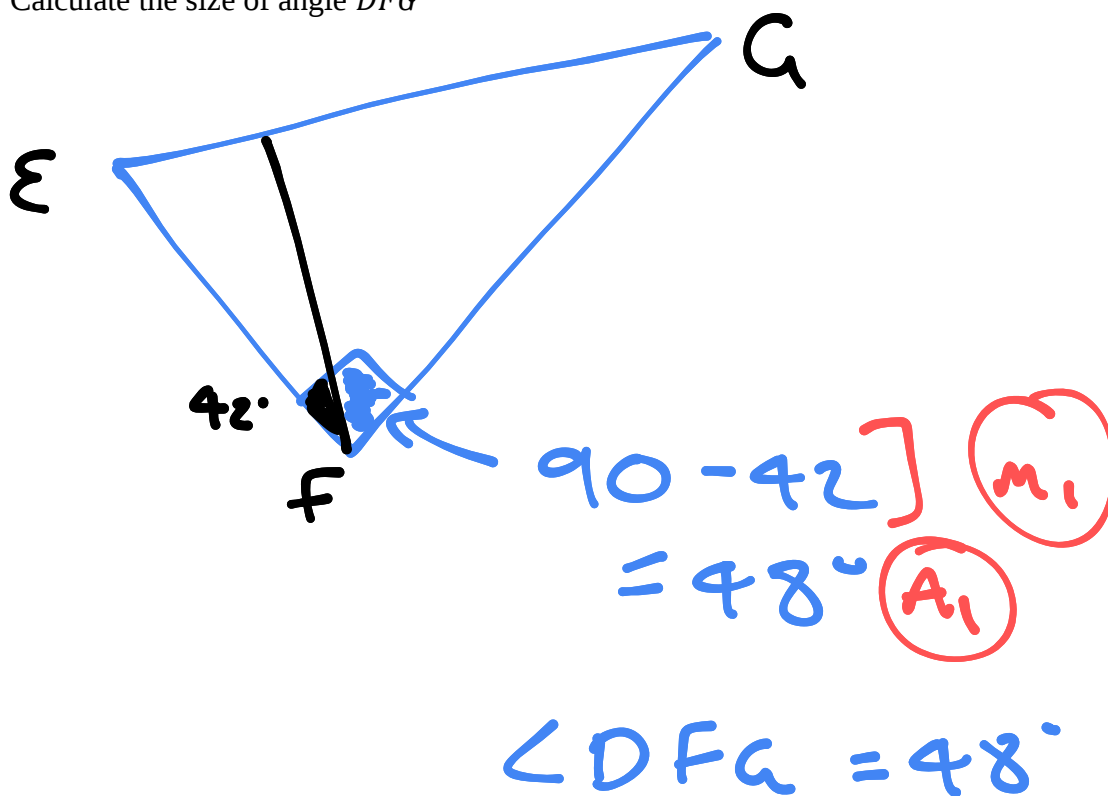
D, E, F and G are points on a circle, centre O

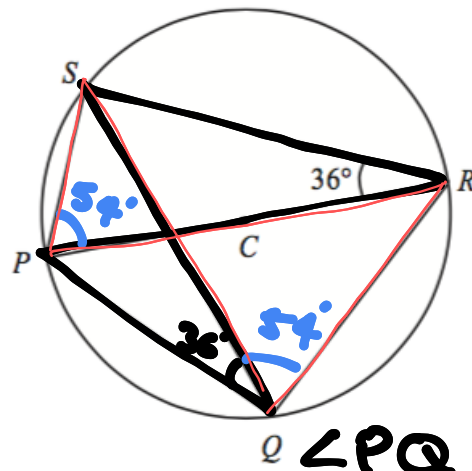


EOG is a diameter of the circle.

Angle $EGD = 42^\circ$

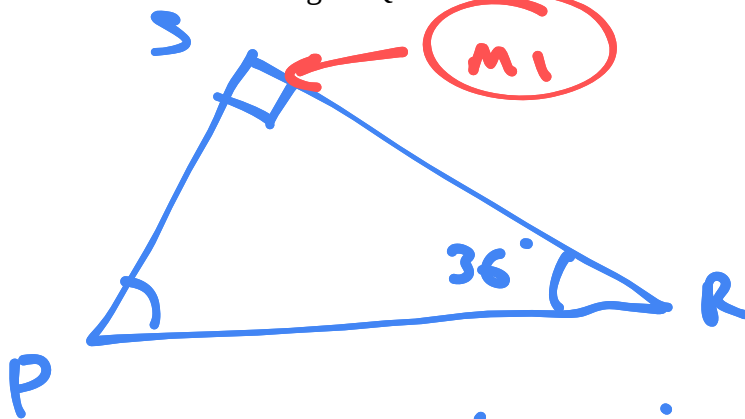
Calculate the size of angle DFG



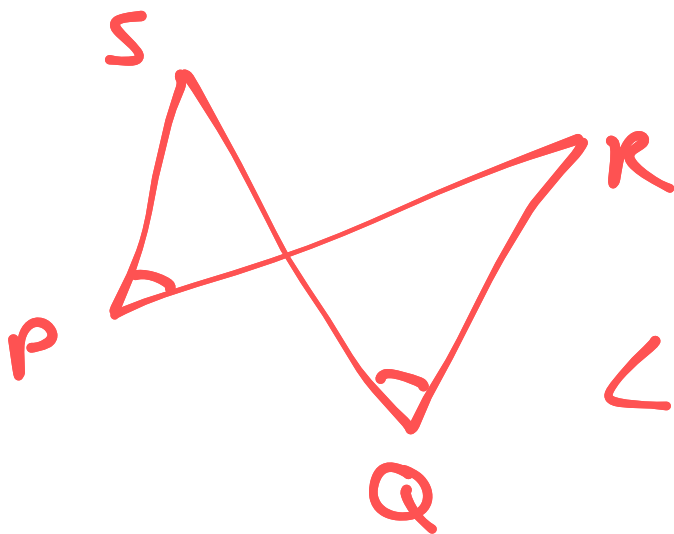


P, Q, R and S are points on a circle, centre C .
 PCR is a straight line.
 Angle $PRS = 36^\circ$.

Calculate the size of angle RQS .



$$180^\circ - 90^\circ - 36^\circ = 54^\circ \quad (A_1)$$



$$\angle RQS = 54^\circ \quad (A_1)$$

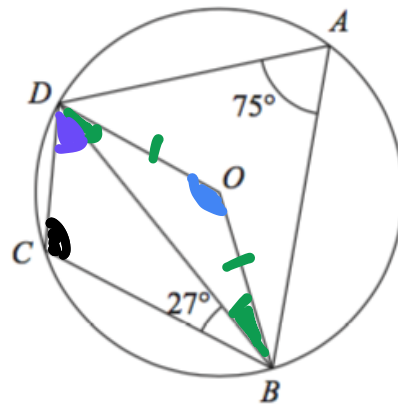


Diagram NOT
accurately drawn

A, B, C and D are points on a circle, centre O .

Angle $DAB = 75^\circ$

Angle $DBC = 27^\circ$

Work out the size of angle ODC .

$$180 - 75 = 105^\circ \quad \text{M1}$$

$$\angle DCB = 105^\circ$$

$$\angle DOB = 150$$

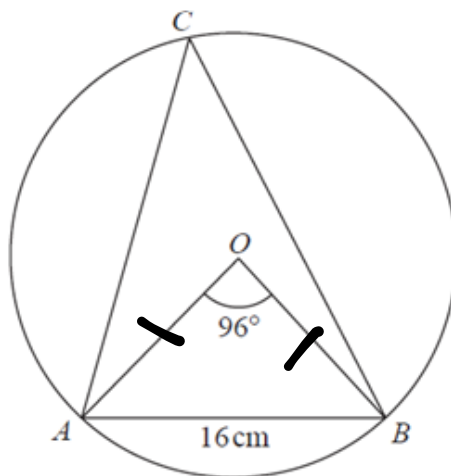
$$\angle ODB = \frac{180 - 150}{2} = 15^\circ \quad \text{M1}$$

$$\angle CDB = 180 - 105 - 27 = 48^\circ \quad \text{M1}$$

$$48^\circ + 15^\circ = 63^\circ \quad \text{A1}$$

(4 marks)

The diagram shows a circle with centre O .
The points A , B and C lie on the circle.



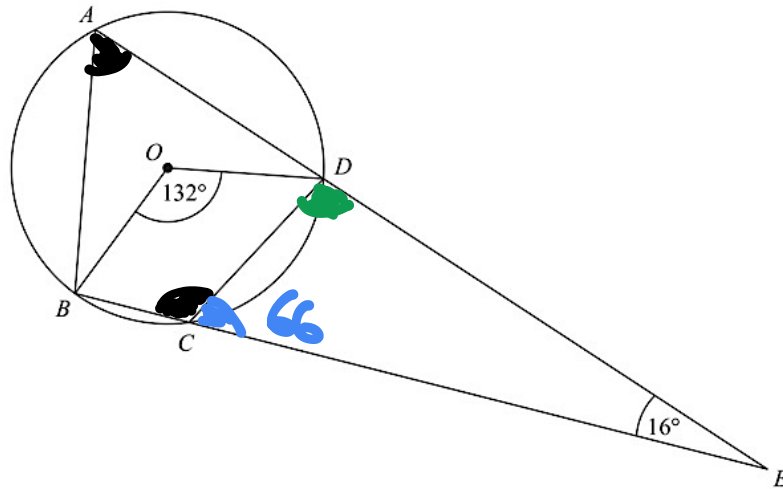
$AB = 16\text{cm}$

Work out the radius of the circle.
Give your answer correct to 3 significant figures.

$$r = \frac{8}{\sin 48}$$

$$r = 10.8 \text{ cm}$$

A , B , C and D are points on the circumference of a circle, centre O .
 ADE and BCE are straight lines.



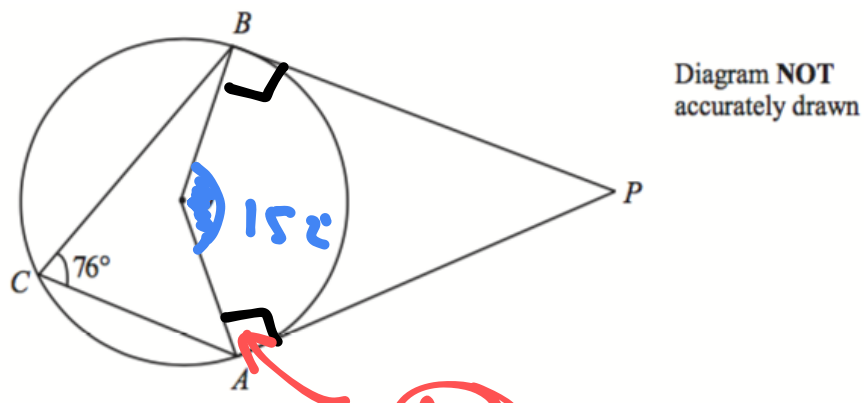
Work out the size of angle CDE .

$$\angle BAD = \frac{132}{2} = 66^\circ \quad (M1)$$

$$\angle BCD = 180 - 66 = 114^\circ \quad (M1)$$

$$180 - 114 = 66 = \angle DCE \quad (M1)$$

$$\begin{aligned} \angle CDE &= 180 - 66 - 16 \\ &= 98^\circ \quad (A1) \end{aligned}$$



A, B and C are points on a circle, centre O.
 Angle $ACB = 76^\circ$.
 PA and PB are tangents to the circle.

Calculate the size of angle APB.

$$\angle BOA = 152^\circ \quad (m_1)$$

$$\underbrace{360 - 152 - 90 - 90}_{(m_1)} = 22^\circ \quad (A_1)$$

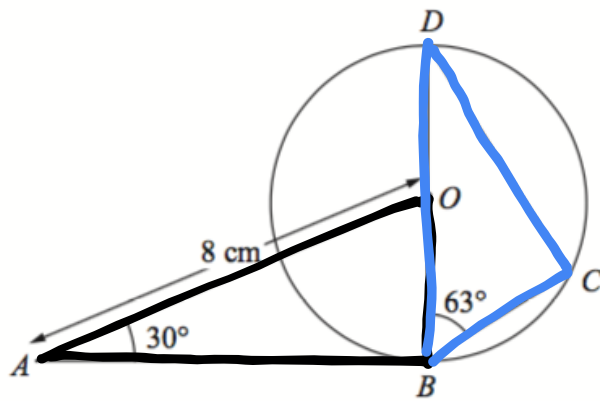


Diagram NOT
accurately drawn

B , C and D are points on a circle, centre O .

BOD is a diameter of the circle.

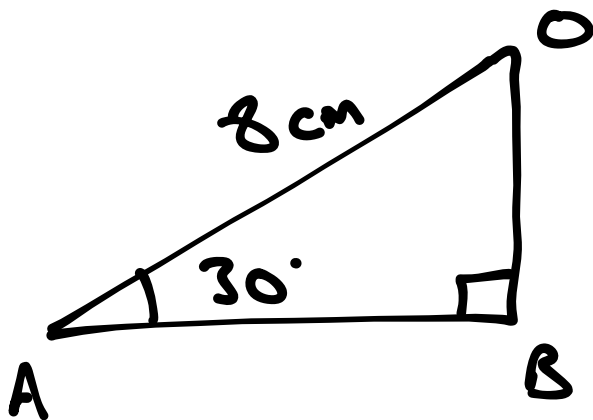
AB is the tangent to the circle at B .

$AO = 8$ cm. Angle $BAO = 30^\circ$ Angle $CBD = 63^\circ$

Calculate the length of BC .

Give your answer correct to 3 significant figures.

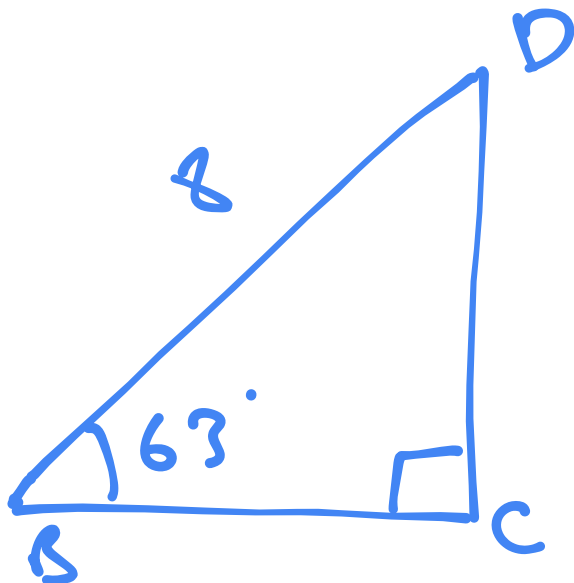
S H



(M1)

$$OB = \sin 30 \times 8$$

$$= 4 \quad (A1)$$



(M1)

$$BC = 8 \times \cos 63^\circ$$

$$BC = 3.63 \text{ cm}$$

(A1)

(4 marks)

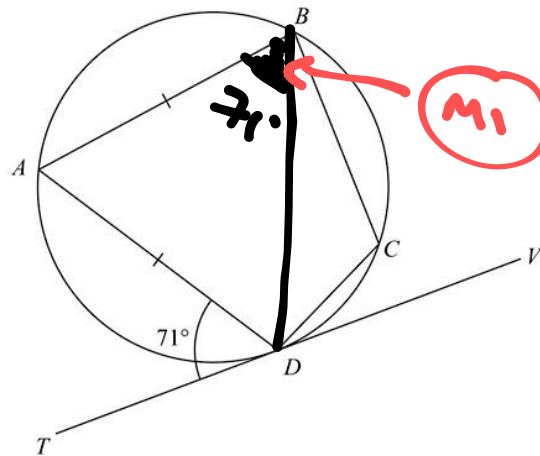


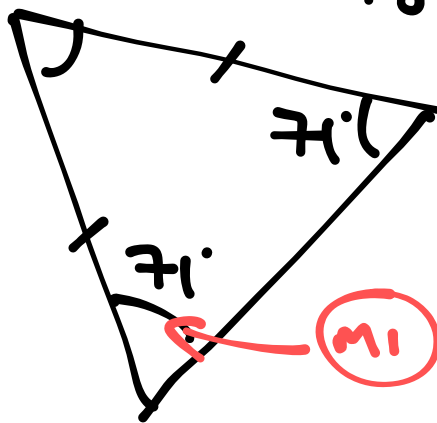
Diagram NOT
accurately drawn

A, B, C and D are points on a circle.
 TDV is the tangent to the circle at D .

$AB = AD$

Angle $ADT = 71^\circ$

Work out the size of angle BCD .

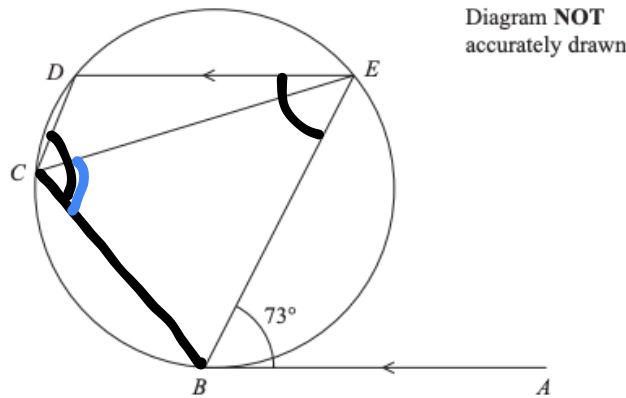


$$180 - 71 - 71 = 38^\circ \quad \text{M1}$$

$$= \angle BAD$$

$$\angle BCD = 180 - 38 = 142^\circ \quad \text{A1}$$

M1



B , C , D and E are points on a circle.

AB is the tangent at B to the circle.

AB is parallel to ED .

Angle $ABE = 73^\circ$

Work out the size of angle DCE .

$$\angle BED = 73^\circ \quad (M1)$$

$$\angle BCE = \angle ABE = 73^\circ \quad (M1)$$

ALTERNATE SEGMENT
THEOREM

$$\angle BCD = 180 - 73 = 107^\circ \quad (M1)$$

OPP. ANGLES IN A CYCLIC
QUADRILATERAL

$$\angle DCE = 107 - 73 = 34 \quad (A1)$$

(5 marks)

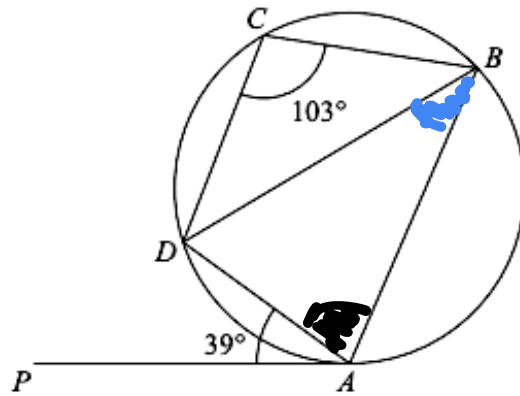


Diagram NOT
accurately drawn

A, B, C and D are points on a circle.

PA is a tangent to the circle.

Angle $PAD = 39^\circ$

Angle $BCD = 103^\circ$

Calculate the size of angle ADB .

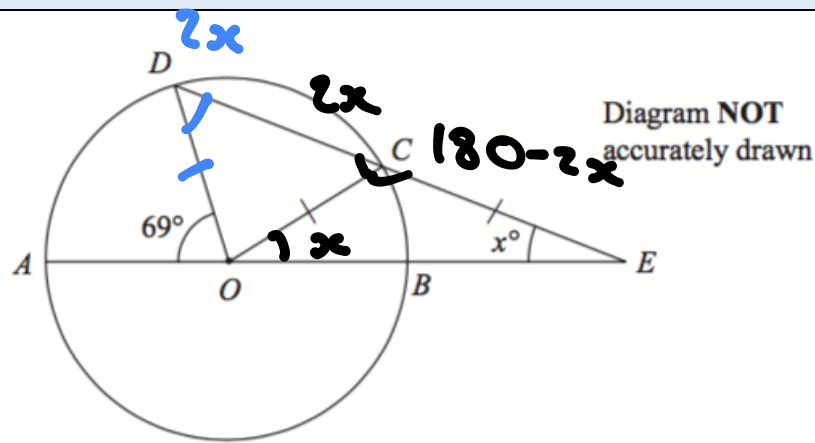
$$\angle BAD = 180 - 103 = 77^\circ \quad (M_1) \quad (A_1)$$

OPP. ANGLES IN A
CYCLIC QUADRILATERAL

$$\angle ABD = 39^\circ \quad (M_1)$$

ALTERNATE SEGMENT
THEOREM

$$180 - 77 - 39 = \underline{64^\circ} \quad (A_1) \quad (A_1)$$



A, B, C and D are points on a circle, centre O.

AOBE and DCE are straight lines.

$CO = CE$.

Angle $AOD = 69^\circ$

Angle $CEO = x^\circ$

Calculate the value of x .

$$\angle OCE = 180 - 2x \quad (M1)$$

$$\begin{aligned} \angle OCD &= 180 - (180 - 2x) \\ &= 2x \quad (M1) \end{aligned}$$

$$\angle ODC = 2x \quad (M1)$$

$$\begin{aligned} \angle COD &= 180 - 2x - 2x \\ &= 180 - 4x \quad (M1) \end{aligned}$$

$$69 + (180 - 4x) + x = 180 \quad (M1)$$

$$69 = 3x \quad \therefore x = 23 \quad (A1) \quad (6 \text{ marks})$$

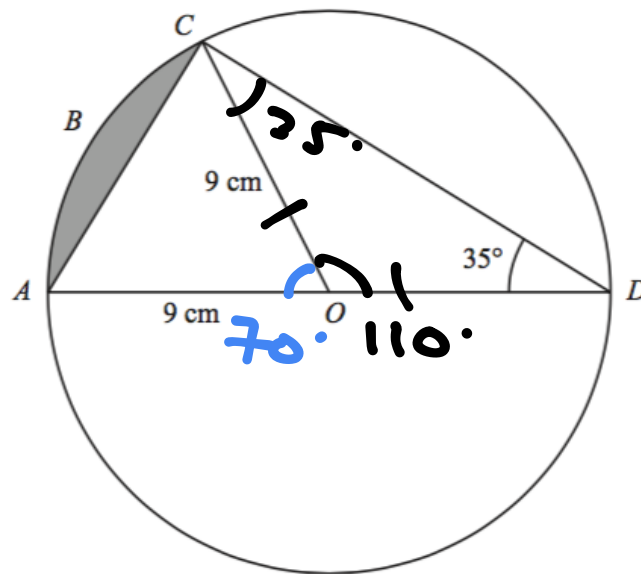


Diagram NOT
accurately drawn

AOD is a diameter of a circle, with centre O and radius 9 cm.

ABC is an arc of the circle.

AC is a chord.

Angle ADC = 35°

Calculate the area of the shaded segment.

Give your answer correct to 3 significant figures.

$$\pi \times 9^2 \times \frac{70}{360} = 49.48$$

$$180 - 35 - 35 = 110$$

$$180 - 110 = 70$$

$$\frac{1}{2} \times 9 \times 9 \times \sin 70 = 38.057$$

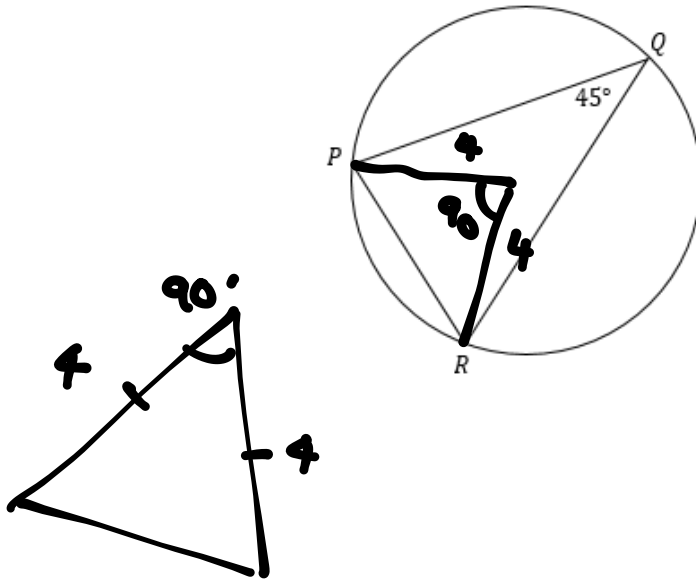
$$49.48 - 38.057 = 11.4 \text{ cm}^2$$

QUESTIONS FROM MATHEMATICAL COMPETITIONS

Q1.

AtoZrevision.com

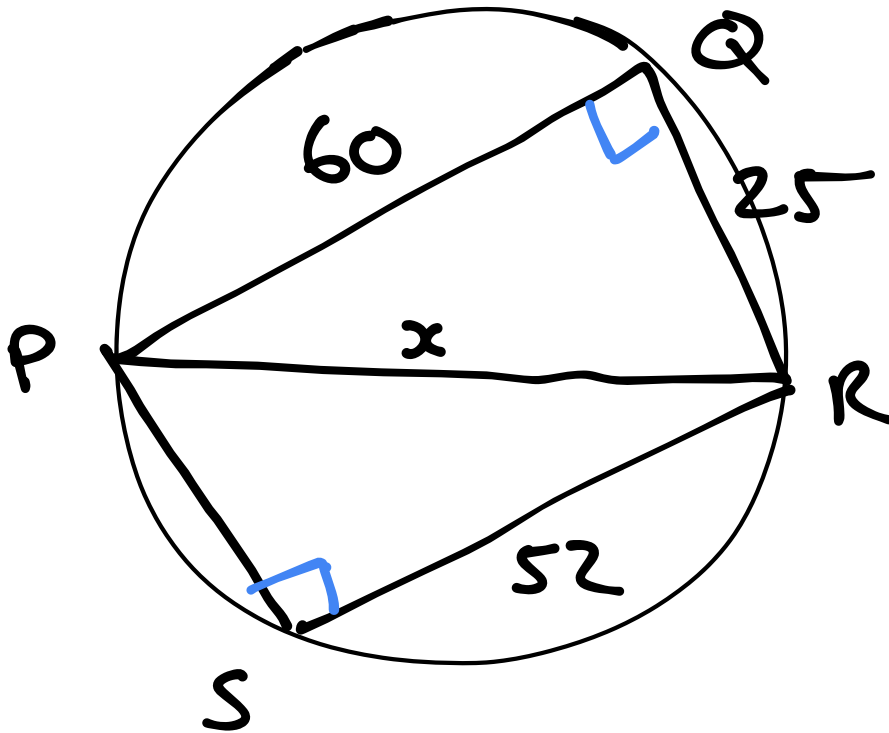
P, Q and R are points on the circumference of a circle with radius 4cm. $\angle PQR = 45^\circ$. What is the exact length of chord PR ?



USING COSINE RULE :

$$PR = \sqrt{4^2 + 4^2 - (2 \times 4 \times 4 \cos 90)} \\ = 4\sqrt{2}$$

$PQRS$ is a quadrilateral inscribed in a circle of which PR is a diameter. The lengths of PQ , QR and RS are 60, 25 and 52 respectively. What is the length of SP ?



$$SP^2 = x^2 - 52^2$$

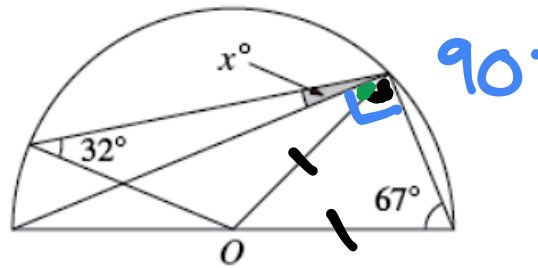
$$x^2 = 60^2 + 25^2 = 4225$$

$$\therefore SP^2 = 4225 - 52^2$$

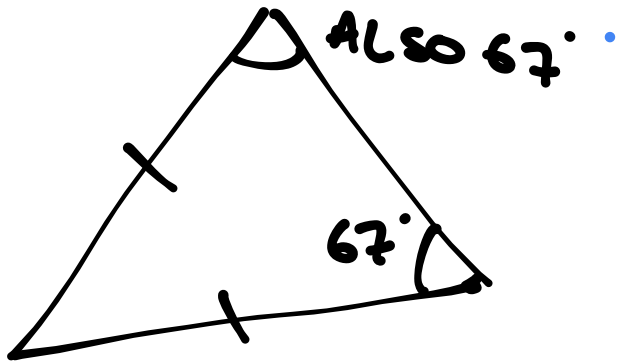
$$SP^2 = 1521$$

$$\therefore SP = 39$$

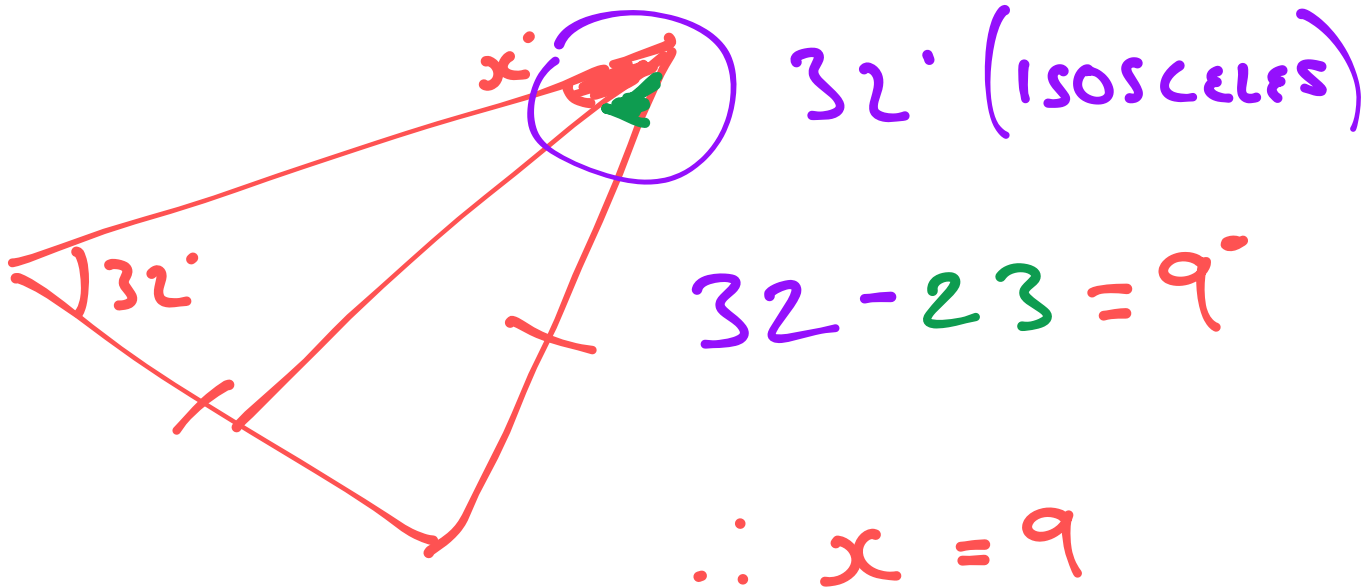
The diagram shows a semicircle with centre O .



Two of the angles are given. What is the value of x ?



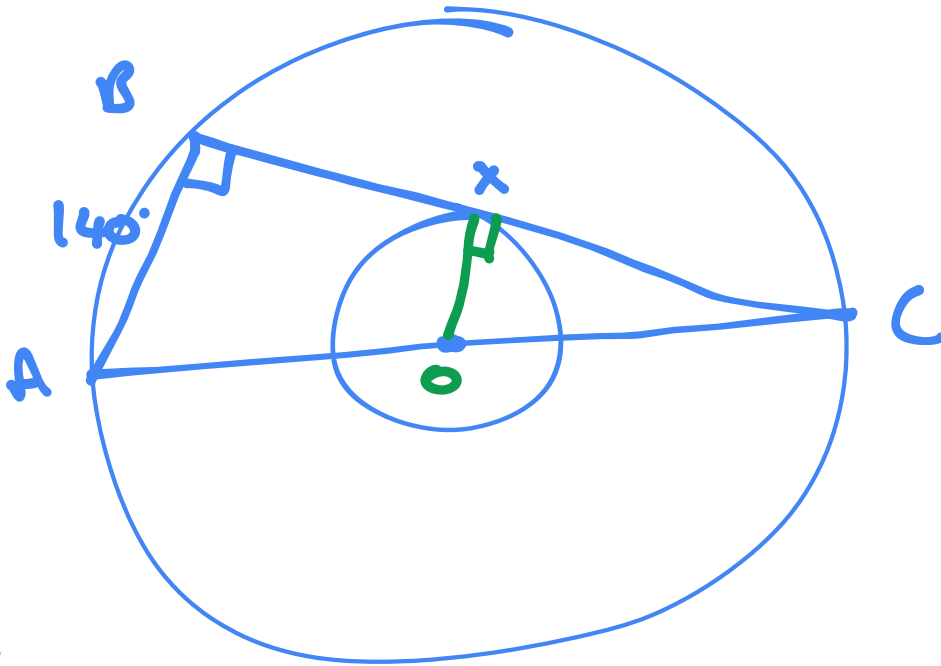
$$90 - 67 = 23$$



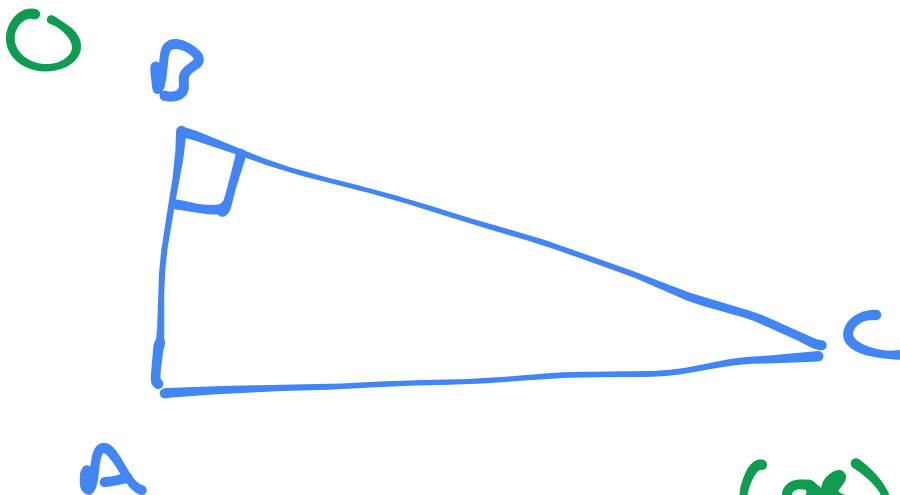
The radii of two concentric circles are in the ratio 1:3.

AC is a diameter of the larger circle. BC is a chord of the larger circle and is tangent to the smaller circle. AB has length 140.

What is the radius of the larger circle?



IS SIMILAR TO:



$$S.F. = 2$$

AS

$$CO : CA \\ 1 : 2$$

(or) $OX : AB = 1 : 2$

$$\therefore OX = 70 \left(\frac{1}{2} \text{ of } 140 \right) = r \quad r \times 3 = \underline{\underline{210}}$$